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THE AHP PHENOMENON OF RANK REVERSAL DEMYSTIFIED

ABSTRACT

Probably the most heated debate about the validity of AHP and ultimately its unclarified phenomenon is its ability to reverse ranks of the alternatives. The objective of this study is to question the evidence of AHP rank reversal phenomenon presented by Belton and Gear as well to extend this study conclusions on other rank reversal examples. The evidence of the research indicates that priority vectors derived from both consistent and inconsistent Pairwise Comparison Matrices are fuzzy and should not be considered as set but only as estimated with certain level of probability. Hence, any evidence showing rank reversal in the AHP models which is based on assumptions about their determined value should be considered as erroneous.

Keywords: AHP, rank reversal phenomenon, Monte Carlo simulations

1. Introduction

The success of AHP is probably due to the fact that Saaty [1] proposed a complete solution for a decision maker (DM) i.e. a ranking calculation algorithm, an inconsistency index as a method of determining data quality and a hierarchical model allowing DMs to handle multiple criteria [2–5]. However, although popular as the easy to use decision making support tool, the concept of AHP revealed in time a few drawbacks which undermined its validity from the perspective of Multi Attribute Utility Theory (MAUT) as well Multi Attribute Value Theory (MAVT). Probably the most heated debate about the validity of AHP and ultimately its unclarified phenomenon is its ability to reverse ranks of the alternatives. However, even the creator of AHP stated once that improving consistency does not mean getting an answer closer to the 'real' life solution. It can be illustrated on the following example. Considered is the true priority vector w (denoting true weights of examined alternatives) i.e. w=[7/20, 1/4, 1/4, 3/20] and $\boldsymbol{A}(w)$ derived from that w, which can be presented as follows:

$$\begin{bmatrix} 1 & 7/5 & 7/5 & 7/3 \\ 5/7 & 1 & 1 & 5/3 \\ 5/7 & 1 & 1 & 5/3 \\ 3/7 & 3/5 & 3/5 & 1 \end{bmatrix}$$

Then, $\boldsymbol{R}(x)$ is considered which is produced by a hypothetical DM. It is assumed that DM is very trustworthy and is able to express judgments very precisely at the same time being still somehow limited by the necessity of expressing judgments on a scale (the example utilizes Saaty's scale). In this scenario, entries of $\boldsymbol{A}(w)$ are rounded to Saaty's scale and the entries are made reciprocal – the principal condition for Pairwise Comparison Matrices (PCM) applied in the AHP:

$$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 1 & 1 & 1 & 2 \\ 1 & 1 & 1 & 2 \\ 1/2 & 1/2 & 1/2 & 1 \end{bmatrix}$$

It should be noted that $\boldsymbol{R}(x)$ is perfectly consistent. **Table 1** presents selected values of the performance measures i.e. CI_{REV} – Saaty's Consistency Index (CI) and CI_{GM} – Aguaron & Moreno-Jimenez's CI for $\boldsymbol{R}(x)$ together with w_E derived from $\boldsymbol{R}(x)$; Absolute Average Errors AAE given as $AAE = \frac{1}{n}\sum_{i=1}^{n}|w_i - w_{Ei}|$ between w_E and w for the case; Spearman Rank Correlation Coefficients (SRCs) among w_E and w for the case.

Table 1 – Values of CI_{REV} and CI_{GM} as well proposed quality characteristics of w estimates – w_E, derived from $\boldsymbol{R}(x)$ with application of the REV and GM method

PM(*)	ESTIMATES – w_E	PERFORMANCE MEASURES		
		CI_{PM}	*AAE*	*SRC*
REV	$[0.285714, 0.285714, 0.285714, 0.142857]^T$	0.0	0.0357143	0.8164966
GM	$[0.285714, 0.285714, 0.285714, 0.142857]^T$	0.0	0.0357143	0.8164966

(*) PM stands for prioritization method

On the bases of similar examples it is intended to examine and question the correctness of the evidence provided by Belton and Gear concerning rank reversal phenomenon.

2. Literature Review

The phenomenon of rank reversal was illustrated by Belton and Gear [6,7]. The authors proposed their approach to avoid the rank reversal, called *B-G modified AHP*. Saaty and Vargas [8] presented the counterexample where the *B-G modified AHP* was also subject to the rank reversal phenomenon. Schoner and Wedley [9] proposed a modified AHP method, called referenced AHP, to avoid rank reversal. Barzilai and Golany [10] showed that no normalization method can prevent rank reversal. Barzilai and Lootsma [11] proposed the multiplicative AHP method for avoiding the rank reversal but Vargas [12] presented an example showing that the multiplicative AHP is invalid. The peculiarity of the problem inclined a few more comprehensive literature reviews on this subject e.g. [13,14].

3. Research Objective

The objective of this study is to question the evidence of AHP rank reversal phenomenon presented by Belton and Gear [7]. Its intent is to show via computer simulations that the example presented by these authors is ambiguous in its nature hence it should be ignored.

4. Research Design/Methodology

The examination involves Monte Carlo simulations. The simulation algorithm is designed in a way it is presented in the Appendix (Fig.1). The examination results are the effect of various scenarios applied to the simulation process which reflects both human judgment errors during pairwise comparisons and technical errors embedded in the AHP .

5. Limitations

Given the reality of our physical world, no study is perfect. We simulate different situations related to various sources of the PCM inconsistency. Fundamentally, the inconsistency commonly results from errors caused by the nature of human judgments and errors due to the technical realization of the comparison procedure i.e. rounding errors and errors resulting from the forced reciprocity requirement. Nature of human judgments can be represented as the realization of some random process in accordance with the assumed probability distribution of the perturbation factor e.g. uniform, gamma, truncated normal

and log-normal. As this is only a stochastic process generated by the computer it is the main limitation of this study.

6. Conclusions

The evidence of the examination indicates that Priority Vectors derived from both consistent and inconsistent Pairwise Comparison Matrices are fuzzy and should not be considered as set but only as estimated with certain level of probability. Hence, any evidence showing rank reversal in the AHP models which is based on assumptions about their determined value should be considered as erroneous.

7. Key References – Appendix 1

[1] T.L. Saaty, The Analytic Hierarchy Process: Decision Making in Complex Environments, in: R. Avenhaus, R.K. Huber (Eds.), Quant. Assess. Arms Control Math. Model. Simul. Anal. Arms Control Probl., Springer US, Boston, MA, 1984: pp. 285–308. https://doi.org/10.1007/978-1-4613-2805-6_12.

[2] T.L. Saaty, Decision making with the analytic hierarchy process, Int. J. Serv. Sci. 1 (2008) 83. https://doi.org/10.1504/IJSSCI.2008.017590.

[3] H. Zhao, L. Yao, G. Mei, T. Liu, Y. Ning, A Fuzzy Comprehensive Evaluation Method Based on AHP and Entropy for a Landslide Susceptibility Map, Entropy. 19 (2017) 396. https://doi.org/10.3390/e19080396.

[4] G. Feng, S. Lei, Y. Guo, B. Meng, Q. Jiang, Optimization and Evaluation of Ventilation Mode in Marine Data Center Based on AHP-Entropy Weight, Entropy. 21 (2019) 796. https://doi.org/10.3390/e21080796.

[5] J. Hodicky, G. Özkan, H. Özdemir, P. Stodola, J. Drozd, W. Buck, Analytic Hierarchy Process (AHP)-Based Aggregation Mechanism for Resilience Measurement: NATO Aggregated Resilience Decision Support Model, Entropy. 22 (2020) 1037. https://doi.org/10.3390/e22091037.

[6] V. Belton, T. Gear, The legitimacy of rank reversal--A comment, Omega. 13 (1985) 143–144. https://ideas.repec.org/a/eee/jomega/v13y1985i3p143-144.html (accessed May 19, 2021).

[7] V. Belton, T. Gear, On a short-coming of Saaty's method of analytic hierarchies, Omega. 11 (1983) 228–230. https://ideas.repec.org/a/eee/jomega/v11y1983i3p228-230.html (accessed January 17, 2019).

[8] T.L. Saaty, L.G. Vargas, The legitimacy of rank reversal, Omega. 12 (1984) 513–516. https://doi.org/10.1016/0305-0483(84)90052-5.

[9] B. Schoner, W.C. Wedley, Ambiguous Criteria Weights in AHP: Consequences and Solutions*, Decis. Sci. 20 (1989) 462–475. https://doi.org/10.1111/j.1540-5915.1989.tb01561.x.

[10] J. Barzilai, B. Golany, Ahp Rank Reversal, Normalization And Aggregation Rules, INFOR Inf. Syst. Oper. Res. 32 (1994) 57–64. https://doi.org/10.1080/03155986.1994.11732238.

[11] J. Barzilai, F.A. Lootsma, Power Relations and Group Aggregation in the Multiplicative AHP and SMART, J. Multi-Criteria Decis. Anal. 6 (1997) 155–165. https://doi.org/10.1002/(SICI)1099-1360(199705)6:3<155::AID-MCDA131>3.0.CO;2-4.

[12] L.G. Vargas, Comments on Barzilai and Lootsma: Why the Multiplicative AHP Is Invalid: A Practical Counterexample, J. Multi-Criteria Decis. Anal. 6 (1997) 169–

170. https://doi.org/10.1002/(SICI)1099-1360(199705)6:3<169::AID-MCDA134>3.0.CO;2-P.
[13] H. Maleki, S. Zahir, A Comprehensive Literature Review of the Rank Reversal Phenomenon in the Analytic Hierarchy Process, J. Multi-Criteria Decis. Anal. 20 (2013) 141–155. https://doi.org/10.1002/mcda.1479.
[14] T.L. Saaty, L.G. Vargas, R. Whitaker, Addressing with Brevity Criticisms of the Analytic Hierarchy Process, Int. J. Anal. Hierarchy Process. 1 (2009). https://doi.org/10.13033/ijahp.v1i2.53.

8. Simulation Algorithm – Appendix B

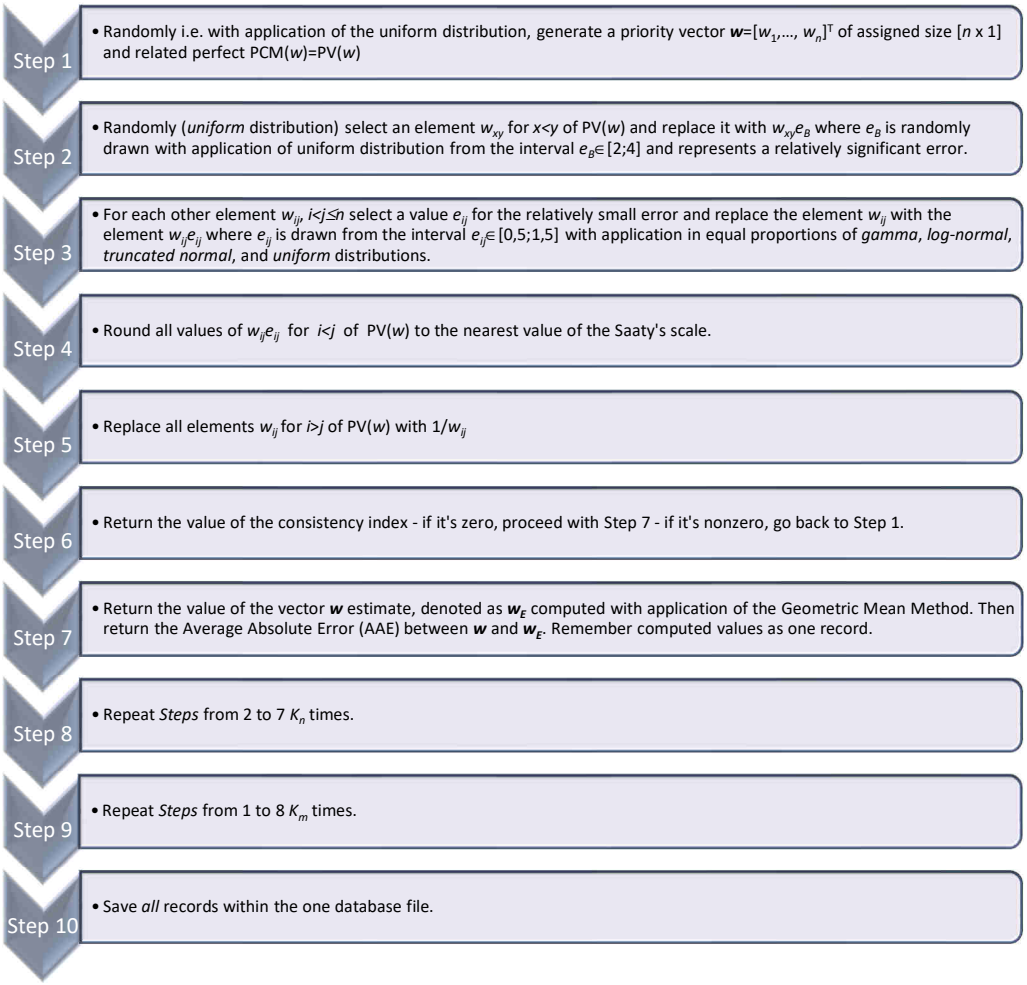

Fig. 1. Simulation algorithm applied for the research