ON THE SIMILARITY AMONG PRIORITY DERIVING METHODS FOR THE AHP

ABSTRACT

There are tens of priority deriving methods for the AHP although the genuine method is commonly applied i.e. the Principal Right Eigenvalue Method (REV). It is known that when decision makers (DM) are consistent with their pairwise judgments about various decision options all available methods result in the same priority vector (PV). However, when DM judgments are inconsistent, and their preferences toward alternative problem solutions are not cardinally transitive, the results usually vary. The research compares a few selected prioritization methods from the perspective of their ranking credibility which is evaluated with the application of a few available statistical measures i.e. Mean Average Absolute Deviation (MAAD), Mean Spearman Rank Correlation Coefficient (MSRC), and Mean Pearson Correlation Coefficient (MPCC). These measures designate the difference between priority vectors estimation quality from the perspective of the selected priority deriving methods. The examined estimates refer to the inconsistency of the Pairwise Comparison Matrices (PCMs) which are obtained during pairwise judgment simulation process. Fundamental considerations are accompanied by Monte Carlo experiments designed for a hypothetical three levels AHP framework. The examination results show the discrepancy among examined prioritization methods from the perspective of their quality.

Keywords: pairwise judgments, ranking, prioritization, REV, LLSM, LUA, SNCS, AHP, Analytic Hierarchy Process.

1. Introduction

Creating a ranking based on comparing alternatives in pairwise mode was already known in the Middle Ages. Probably the first work on this subject was by Ramon Lull (Colomer, 2013), who described election process based on the mutual alternatives comparisons. Over time, other research on the pairwise comparison method appeared e.g. studies on electoral systems, such as the Condorcet method (Condorcet, 1785; Young, 1988) and the Copeland method (Saari & Merlin, 1996), and many other on the social choice and welfare systems (Arrow et al., 2011). In time, alternatives began to be compared quantitatively which was initially connected with the need to compare psychophysical stimuli (Fechner, 1860; Thurstone, 1927). This path was later developed (David, 1988) and used in various forms for different objectives, including economics (Peterson & Brown, 1998), consumer research, psychometrics, health care (Kakiashvili et al., 2017) and others. Thanks to Saaty and his seminal paper (Thomas L Saaty, 1977) in which he defined AHP (Analytic Hierarchy Process), comparing alternatives in pairwise mode began to be considered basically as a multi-criteria decision-making method. The undisputable success of the AHP is probably due to the fact that Saaty proposed a complete solution including a ranking calculation algorithm, an inconsistency index as a method of determining data quality, and a hierarchical model allowing Decision Makers (DM) to handle multiple criteria (Thomas L. Saaty, 2008). However, different research studies concerning the pairwise comparison method has resulted in many priority deriving algorithms, and also various inconsistency measures which are beyond the scope of the research. Probably the two most popular algorithms for deriving priorities are REV (Principal Right Eigenvalue Method proposed by Saaty) and LLSM (Logarithmic Least Squared Method known also as Geometric Mean Method devised by Crawford and Williams (Crawford & Williams, 1985a, 1985b)). Less known, but very attractive are LUA (Logarithmic Utility Approach Method suggested by Kazibudzki and Grzybowski (Kazibudzki & Grzybowski, 2013)) and SNCS (Simple Normalized Column Sum Procedure promoted by Choo and Wedley (Choo & Wedley, 2004)). It is easy to verify that, for consistent matrices, all methods lead to the same solution. However, in the case of inconsistent matrices, the resulting rankings differ from each other. This research is a part of the discussion on the properties of various priority deriving algorithms applied during pairwise judgments within the AHP. Despite the large number of publications on the topic, the AHP fundamentals are still inspiring and challenging. It seems prerequisite that pairwise judgments process within the AHP could transfer to credible measures of DM preferences.

2. The Research Objective

Deriving true priority vectors from intuitive pairwise judgments of decision makers is a crucial issue within the multiple criteria decision making methodology called Analytic Hierarchy Process (AHP). The most popular procedure in the ranking process, constitutes the REV. The standard AHP applications commonly apply the REV because as it was derived mathematically it provides the only right solution in this process. The objective of the proposed scientific research is to examine if this statement can be considered as experimentally verified. Thus, it was decided to apply Monte Carlo methodology for this purpose. However, rather than simulate and analyze results for singular Pairwise Comparison Matrices, as it has been done so far by many other authors, it was decided to design Monte Carlo simulations and analyze their outcome for a simple decision model within the most common AHP framework. It is assumed, that the AHP framework consist of three levels: goal, criteria and alternatives, which is supposed to reflect the hypothetic case of real decisional problem. Then, the simulation results for selected prioritization methods are compared. The examination results are the effect of various scenarios applied to the simulation process and reflect human judgment errors during pairwise comparisons.

3. Research Design/Methodology

The first step in using AHP is to develop a hierarchy by breaking a problem down into its primary components. The basic AHP model includes the goal (a statement of the overall objective), criteria (the factors that should be considered in reaching the ultimate decision) and alternatives (the feasible alternatives that are available to achieve said ultimate goal). The most common and basic AHP structure consists of a goal-criteria-alternatives sequence. The examination assumes a fundamental three level hierarchy encompassing three criteria and three alternatives under each criterion. The intent of the examination is to evaluate performance of the REV on the background of performance of other selected methods available for the AHP. In order to achieve this objective we are going to proceed with Monte Carlo simulations. However, simulations not commonly known i.e. dedicated only to a singular unit Pairwise Comparison Matrix (PCM). The research simulation scenario will involve the entire AHP framework which is supposed to reflect the hypothetical decisional problem, see the appendix for exemplification.

4. Limitations

Given the reality of our physical world, no study is perfect. In order to compare the accuracy of the estimations obtained by selected priority deriving methods we simulate different situations related to various sources of the PCM inconsistency. Fundamentally, the inconsistency commonly results from errors caused by the nature of human judgments and errors due to the technical realization of the comparison procedure i.e. rounding errors and errors resulting from the forced reciprocity requirement. Nature of human judgments can be represented as the realization of some random process in accordance with the assumed probability distribution of the perturbation factor e.g. uniform, gamma, truncated normal and log-normal. As this is only a stochastic process generated by the computer it is the main limitation of the proposed examination.

5. Conclusions

It seems quite reasonable to examine performance of other priority deriving methods that can successfully operate within the AHP. It also seems very reasonable to make the effort and strive to reduce consequences of humans imperfection while discovering their preferences with application of different priority deriving methods.

6. Key References

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7. Appendix

In order to clarify the examination framework introduced in this research proposal we are presenting its simplified version as the methodological example. Thus, we take into consideration only technical perturbation of PCMs resulting from rounding errors during application of Saaty's scale and standard requirement within AHP i.e. forced reciprocity. Thus, the following hypothetical model of the AHP framework with three levels (four criteria and four alternatives) is considered:

with respect to the GOAL:

$$\begin{array}{c} \\ C1 \\ C2 \\ C3 \\ C4 \end{array} \begin{array}{c} \begin{array}{cccc} C1 & C2 & C3 & C4 \end{array} \\ \begin{bmatrix} 1 & 1.4 & 3.5 & 1.16667 \\ 0.714286 & 1 & 2.5 & 0.833333 \\ 0.285714 & 0.4 & 1 & 0.333333 \\ 0.857143 & 1.2 & 3 & 1 \end{bmatrix} \end{array} \quad \begin{array}{c} \mathrm{H_G PV_C} \\ \begin{bmatrix} 0.35 \\ 0.25 \\ 0.10 \\ 0.30 \end{bmatrix} \end{array}$$

with respect to criteria C1–C2:

$$\begin{array}{c} \\ A1 \\ A2 \\ A3 \\ A4 \end{array} \begin{array}{c} \begin{array}{cccc} A1 & A2 & A3 & A4 \end{array} \\ \begin{bmatrix} 1 & 1.4 & 2.33333 & 1.4 \\ 0.714286 & 1 & 1.66667 & 1 \\ 0.428571 & 0.6 & 1 & 0.6 \\ 0.714286 & 1 & 1.66667 & 1 \end{bmatrix} \end{array} \quad \begin{array}{c} \mathrm{H_{C2}^{C1} PV_A} \\ \begin{bmatrix} 0.35 \\ 0.25 \\ 0.15 \\ 0.25 \end{bmatrix} \end{array}$$

with respect to criteria C3–C4:

$$\begin{array}{c} \\ A1 \\ A2 \\ A3 \\ A4 \end{array} \begin{array}{c} \begin{array}{cccc} A1 & A2 & A3 & A4 \end{array} \\ \begin{bmatrix} 1 & 0.666667 & 0.285714 & 0.25 \\ 1.5 & 1 & 0.428571 & 0.375 \\ 3.5 & 2.33333 & 1 & 0.875 \\ 4 & 2.66667 & 1.14286 & 1 \end{bmatrix} \end{array} \quad \begin{array}{c} \mathrm{H_{C4}^{C3} PV_A} \\ \begin{bmatrix} 0.10 \\ 0.15 \\ 0.35 \\ 0.40 \end{bmatrix} \end{array}$$

where $\mathrm{H_G PV_A}$, $\mathrm{H_{C2}^{C1} PV_A}$, $\mathrm{H_{C4}^{C3} PV_A}$ denote partial hypothetical PV in the model.

After standard AHP synthesis, the hypothetical total PV (HTPV) is obtained HTPV=$[0.25, 0.21, 0.23, 0.31]^T$. Next, following the simplified examination scenario, each PCM in the presented framework is going to be perturbed. For illustration purpose, only two kinds of distortions are applied i.e. each element of the particular PCM is rounded to Saaty's numerical scale and the PCM is transformed to be reciprocal. Then, on the bases of every distorted PCM and with application of the REV, respective partial PVs (PPV_{REV}) are computed. Finally, the total calculated priority vector ($TCPV_{REV}$) for the exemplary model of the AHP is computed. The model can be presented as follows:

with respect to the GOAL:

$$\begin{array}{c} \\ C1 \\ C2 \\ C3 \\ C4 \end{array} \begin{array}{c} \begin{array}{cccc} C1 & C2 & C3 & C4 \end{array} \\ \begin{bmatrix} 1 & 1 & 3 & 1 \\ 1 & 1 & 2 & 1 \\ 1/3 & 1/2 & 1 & 1/3 \\ 1 & 1 & 3 & 1 \end{bmatrix} \end{array} \quad \begin{array}{c} \mathrm{PPV}_{\mathrm{REV}}^{\mathrm{G}} \\ \begin{bmatrix} 0.304999 \\ 0.276859 \\ 0.113143 \\ 0.304999 \end{bmatrix} \end{array}$$

with respect to criteria C1–C2:

$$\begin{array}{c} \\ A1 \\ A2 \\ A3 \\ A4 \end{array} \begin{array}{c} \begin{array}{cccc} A1 & A2 & A3 & A4 \end{array} \\ \begin{bmatrix} 1 & 1 & 2 & 1 \\ 1 & 1 & 2 & 1 \\ 1/2 & 1/2 & 1 & 1/2 \\ 1 & 1 & 2 & 1 \end{bmatrix} \end{array} \quad \begin{array}{c} \mathrm{PPV}_{\mathrm{REV}}^{\mathrm{C1C2}} \\ \begin{bmatrix} 0.285714 \\ 0.285714 \\ 0.142857 \\ 0.285714 \end{bmatrix} \end{array}$$

with respect to criteria C3–C4:

$$\begin{array}{c} \\ A1 \\ A2 \\ A3 \\ A4 \end{array} \begin{array}{c} \begin{array}{cccc} A1 & A2 & A3 & A4 \end{array} \\ \begin{bmatrix} 1 & 1/2 & 1/4 & 1/4 \\ 2 & 1 & 1/2 & 1/3 \\ 4 & 2 & 1 & 1 \\ 4 & 3 & 1 & 1 \end{bmatrix} \end{array} \quad \begin{array}{c} \mathrm{PPV}_{\mathrm{REV}}^{\mathrm{C3C4}} \\ \begin{bmatrix} 0.0887547 \\ 0.1611320 \\ 0.3550190 \\ 0.3950950 \end{bmatrix} \end{array}$$

After standard AHP synthesis, the following result is obtained $\mathrm{TCPV}_{\mathrm{REV}}$=[0.2034, 0.2336, 0.2316, 0.3315]^T which is different from HTPV=[0.25, 0.21, 0.23, 0.31]^T. Comparing HTPV with its estimate $\mathrm{TCPV}_{\mathrm{REV}}$ earlier mentioned performance measures i.e. MSRC, MPCC and MAAD which reflect estimation quality of the REV can be computed. For the above exemplary values of HTPV and $\mathrm{TCPV}_{\mathrm{REV}}$ these measures are SRCC=0.2, PCC=0.8142, MAD=0.023325. Noticeably, comparison of estimation quality of any priority deriving method available for AHP is possible in this way. The intention of the proposed research is to compare four of them i.e. REV, LUA, LLSM and SNCS.