GROUPING REPRESENTATIVE POINTS IN AHP-FUZZYSORT WITH AGGLOMERATIVE HIERARCHICAL CLUSTERING

ABSTRACT

The AHP-FuzzySort model extends the AHP-Sort II to improve the assignment of alternatives to different classes by using the fuzzy set theory. Both algorithms demand pairwise comparisons among representative points (RP) and profiles, to reduce the number of comparisons. This contribution formalizes in the AHP-FuzzySort model the use of the agglomerative hierarchical clustering (AHC) to group profiles and RP.
Keywords: AHP-FuzzySort, agglomerative hierarchical clustering.

1. Introduction

In multi-criteria decision sorting problems, each alternative is first evaluated according to several criteria, and then sorted into a predefined class. Among other sorting methods, AHP-FuzzySort (Ishizaka, Tasiou, & Martínez, 2020) stands out because it extends the AHP-Sort II and models the classes through fuzzy set theory (Zadeh, 1975). Instead of using the classical disjoint classes, AHP-FuzzySort can classify alternatives into more than one class, with a membership degree for each one. AHP-FuzzySort reduces the number of the pairwise comparisons among RPs and profiles by grouping them and comparing these overlapped groups. Here, it is proposed a formal grouping process in AHP-FuzzySort by using an AHC method (Bouguettaya et al., 2015). It progressively groups the elements to compare according to their similarity, obtaining a dendrogram which shows the progressive grouping of the data.

2. Literature Review

AHPSort (Ishizaka, Pearman, & Nemery, 2012) was first designed to classify alternatives within ordered classes, and extended by AHPSort II (Miccoli & Ishizaka, 2017) for problems with large number of alternatives. Lately, AHP-FuzzySort (Ishizaka, Tasiou, & Martínez, 2020) was proposed as a fuzzy version of the AHPSort II method, which provides a fuzzy representation of the sorting classes and classifies the alternatives according to their corresponding membership degrees. On the other hand, AHC (Bouguettaya et al., 2015) is a clustering approach which gradually merges similar clustering pairs to generate a hierarchy and select a proper number of clusters.

3. Hypotheses/Objectives

The use of AHC in AHP-FuzzySort aims at formalizing the grouping process of RPs and profiles (see Fig.1 in Appendix 9.1) that are not clearly defined yet.

4. Research Design/Methodology

In any AHPSort model first, it is necessary to define the goal, alternatives, criteria, (fuzzy) classes and profiles. Afterwards, the criteria weights are obtained from the classical AHP eigenvalue method. Then, a few RPs well-distributed across the scale of each criterion are selected, which are lately pairwise compared with the profiles. The AHC method is applied

to classify the elements into overlapped clusters (see Fig. 1) and reduce the dimension of the matrices. From these matrices, the local priorities for RPs and profiles are derived by the AHP eigenvalue method and normalizing the steps from one cluster to another, which are employed to compute the local priority of the alternatives. Finally, the local priorities of both profiles and alternatives are aggregated to obtain the global priorities and sort the alternatives. Notice, as in AHP, a consistency ratio is computed for each pairwise comparison matrix. A scheme of the proposal is shown in Fig. 2.

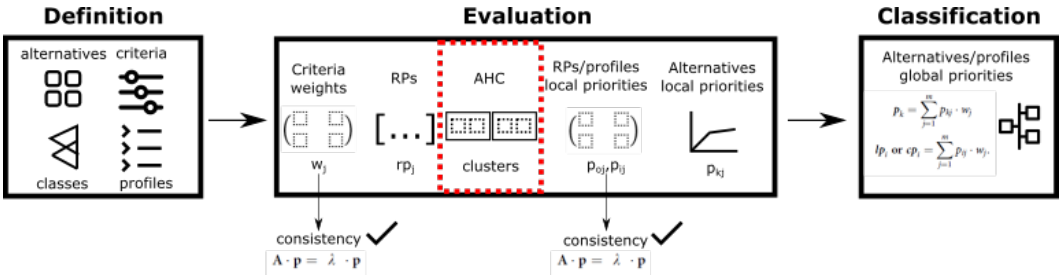

Fig.2: Scheme of the proposed method

5. Data/Model Analysis

See Appendix 9.2.

6. Limitations

For a huge number of RPs and profiles to compare, the AHC may not be efficient, and it can become difficult to determine the correct number of clusters by the dendrogram. However, the number of RPs are usually low to keep the process understandable.

7. Conclusions

This proposal has introduced the use of AHC in AHP-FuzzySort to formalize and automatize the clustering and grouping process of RPs and profiles, resulting in a more efficient version to build and collect the pairwise comparisons required.

8. Key References

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9. Appendix

9.1 Figures

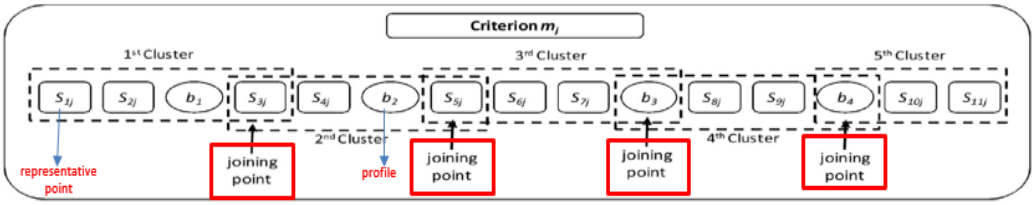

Fig.1: Clustering representative points and profiles

9.2 Illustrative example

Here, we provide an illustrative example in which several countries inside the European Union are evaluated according to the sustainable development goal "Decent work and economic growth" described in Agenda 2030 (https://ec.europa.eu/eurostat/data/database). The criteria to measure the achievement of such a goal are as follows: C1- Real Gross Domestic Product (GDP) per capita, C2- Investment share of GDP by institutional sectors, C3- Young people neither in employment nor in education and training by sex, C4- Employment rate by sex, C5- Long-term unemployment rate by sex, C6- In work at-risk-of-poverty rate, C7- Inactive population due to caring responsibilities by sex, and C8- Resource productivity and domestic material consumption.

In order to classify the different countries, four classes are considered, namely, Total, Moderate, Weak, Null. The RPs for these criteria are compiled in Table 1, which also show the corresponding limiting profiles (LPs).

Table 1. RPs and LPs for the criteria.

	RP1	**RP2**	**RP3**	**RP4**	**RP5**	**RP6**	**LP1**	**LP2**	**LP3**
C1	4500	20294	36088	51882	67676	83470	35000	25000	15000
C2	11,14	16,27	21,41	26,54	31,68	36,82	28	20	12
C3	4,7	9,12	13,54	17,96	22,38	26,8	6	13	20
C4	54,9	60,4	65,9	71,4	76,9	82,4	70	65	60
C5	0,6	4,12	7,64	11,16	14,68	18,2	2	9	16
C6	3,1	6,28	9,46	12,64	15,82	19	5	11	17
C7	1,6	12,38	23,16	33,94	44,72	55,5	8	30	50
C8	67,855	102,83	137,81	172,78	207,76	242,74	200	138	76

For each criterion, RPs and LPs are clustered by an AHC, see Table 2 and Fig. 3.

Table 2. Results of the clustering process.

	Group 1	**Group 2**	**Group 3**
C1	{RP1,LP3,RP2,LP2,LP1,RP3}	{RP3,RP4,RP5}	{RP5,RP6}
C2	{RP1,LP3,RP2,LP2,RP3}	{RP3,RP4,LP1,RP5}	{RP5,RP6}

C3	{RP1,LP1,RP2,LP2,RP3}	{RP3,RP4,LP3,RP5}	{RP5,RP6}
C4	{RP1,LP3,RP2,LP2,RP3,LP1,RP4}	{RP4,RP5}	{RP5,RP6}
C5	{RP1,LP1,RP2}	{RP2,RP3,LP2,RP4}	{RP4,RP5,LP3,RP6}
C6	{RP1,LP1,RP2}	{RP2,RP3,LP2,RP4}	{RP4,RP5,LP3,RP6}
C7	{RP1,LP1,RP2}	{RP1,RP3,LP2,RP4}	{RP4,RP5,LP3,RP6}
C8	{RP1,LP3,RP2}	{RP2,RP3,LP2, RP4, LP1, RP5}	{RP5,RP6}

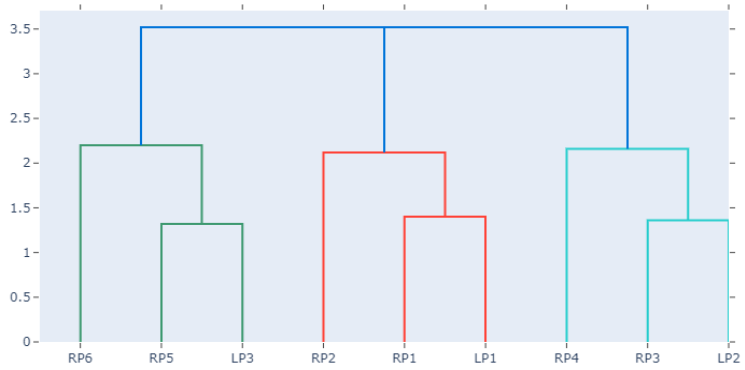

Fig.3: Dendrogram obtained after the AHC for criterion C5.

At this point, note that the classic version of AHP-FuzzySort would require 9×9 matrices containing the pairwise comparison, however applying the AHC method, the expert only needs to provide the pairwise comparisons corresponding to elements of the same cluster. After gathering such pairwise comparisons matrices, the local priorities of the RPs and LPs are normalized for each criterion. Then, the local priorities of each alternative are computed by using the RPs and LPs local priorities. Afterwards, the global priorities for the LPs and alternatives are computed from their local priorities. Finally, the global priorities of the alternatives are evaluated according to the global priorities of the LPs to sort the alternatives (see Fig. 4).

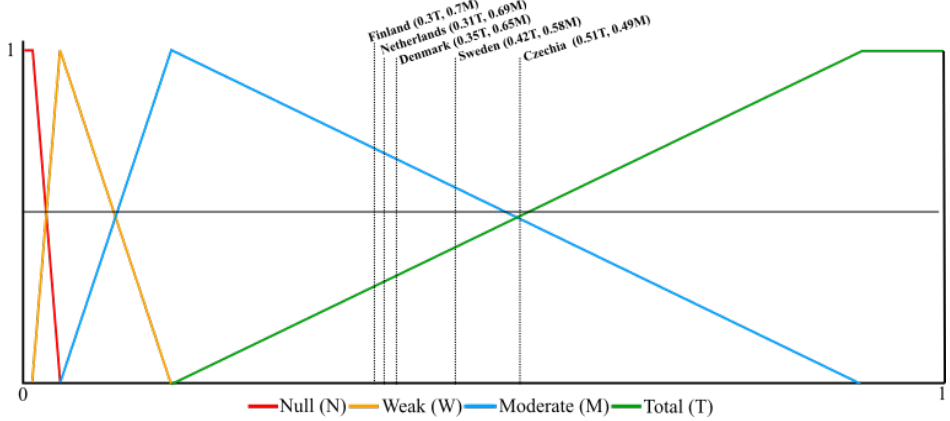

Fig 4: Membership degrees of Top5 alternatives and classification.