SIGNIFICANCE OF THE ORDER OF PAIRWISE COMPARISONS IN AHP: AN UPDATE

ABSTRACT

The paper outlines the authors' latest research on the impact of the order of pair-wise comparisons (PC) upon AHP expert session results. The authors suggest a specific PC order, based on initial rough ranking of compared objects. According to obtained empirical results, in most cases, this PC order makes priority vectors more credible in the eyes of the experts, and pair-wise comparison matrices (PCM) – more consistent. The suggested PC order provides the basis for potential reduction of the number of PC, required from the experts, without significant impact upon the credibility of the session results. The suggested approach is used to modify a well-known combinatorial (spanning tree enumeration) method of priority vector calculation.

Keywords: AHP, pair-wise comparison matrix, spanning tree, ranking, consistency ratio.

1. Introduction

The overall goal of the current study is to reduce the labor-intensity and computational complexity of PC-based methods, such as AHP, without compromising the credibility of expert session result. For this purpose, the authors have suggested a specific order of PC comparisons in AHP, based on preliminary ranking of compared objects. While devising this PC order, they used the outcomes of earlier studies in cognitive psychology.

2. Literature Review

One of the first studies of the impact of order of estimates upon estimation results (for tangible measurable criteria) was performed by [Stevens&Galanter, 1957]. [Wedley, 2009] applied ranking-based approach to reduction of PC numbers in AHP. Other famous ranking-based PC approaches include best-worst method [Rezaei, 2015] and "TOP 2" (best-second best). The approach developed in our paper was originally suggested in [Andriichuk et al, 2020]. In the current research we are going to try to further validate it.

3. Hypotheses/Objectives

The study aims to empirically confirm the hypothesis, that expert session results become more accurate, credible, and consistent if objects are presented to the expert for comparison in a specific order. Another important objective is to use the suggested PC order as basis for reduction of the number of PC comparisons in AHP and to reduce the computational complexity of PC aggregation methods, while preserving the level of credibility of expert session results.

4. Research Design/Methodology

Let n compared objects be numbered according to their rank order, as follows: $a_1 \geq a_2 \geq \cdots \geq a_n$. Then, in order to improve the accuracy of judgments and priorities, we suggest using the following sequence of PC of objects: 1st queue: (a_1, a_n); 2nd queue: (a_1, a_{n-1}) or (a_2, a_n); … ; queue $(n-1)$: (a_1, a_2) or (a_2, a_3) or … or (a_{n-1}, a_n). In our experiment, every expert devised a unique AHP model (5 to 7 criteria) and performed 3 rounds of pair-wise comparisons of criteria. Each round featured a specific PC sequence (A – suggested PC order, B – random order, C – opposite to A). After priority calculation, using all 3 sequences, each respondent was presented 3 respective priority vectors and asked to select the most adequate one among them.

5. Data/Model Analysis

In most cases, the experts selected priority vectors obtained based on sequence A as the most adequate ones. Moreover, PCM, obtained based on sequence A, are, generally, more consistent than those, obtained based on other sequences. I. e., in most cases, *CR* values of PCM, obtained based on sequence A, are slightly smaller than those of PCM, obtained based on other two sequences. The approach also allows to reduce PC number. PCs from upper queues are more informative. So, if we follow the suggested PC order, the minimum set of $n-1$ comparisons can be represented by a bi-partite spanning tree graph, where the first (best) object is connected (compared) to objects from $([n/2]+1)$ to n, while the last (worst) object is connected (compared) to objects from 1 to $[n/2]$. It is to this basic set (spanning tree) that next PCs should be added, to ensure redundancy. If we choose to calculate priorities using combinatorial spanning tree enumeration (instead of eigenvector) method, then a spanning tree can be assigned a weight, based on queues, to which the respective PCs belong. The lower the queue, the greater the weight.

6. Limitations

Preliminary rough ranking of compared objects is still a subject of debate. In experiments with real respondents n is limited to 7 ± 2 objects. Within the experiment it was hard to obtain enough data for statistically credible empirical research (hence, often *CR*>10%).

7. Conclusions and further research

An approach to PC comparison ordering in AHP has been suggested. If the suggested order is used, estimation results tend to be more accurate, credible, and consistent. The suggested approach also provides possibilities for reduction of PC number in AHP. The key direction of future research is comparison of the approach with other ranking-dependent PC methods, such as best-worst and TOP 2 (through simulations).

8. Key References

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