Priority Vector Estimation: Consistency, Compatibility, Precision

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This work compares four methods of the AHP priority vector estimation: EM, LS, LN, and RE:

- EM – Eigenproblem Method, proposed in (Saaty T.L., A scaling method for priorities in hierarchical structures, J. of Mathematical Psychology, 15, 1977, 234–281; Saaty T.L., The Analytic Hierarchy Process. McGraw-Hill, New York, 1980);
- LS – Least Squares method, proposed in (Saaty T.L. and Vargas L.G., Comparison of eigenvalue, logarithmic least squares and least squares methods in estimating ratios, Mathematical Modelling, 5, 1984, 309-324; Saaty T.L. and Vargas L.G., Decision Making in Economic, Political, Social and Technological Environment with the Analytic Hierarchy Process. RWS Publications, Pittsburgh, PA, 1994);
- LN – Logarithmic least squares, or multiplicative method (Saaty and Vargas, 1984, 1994; Lootsma F., Scale sensitivity in the multiplicative AHP and SMART, J. of Multi-Criteria Decision Analysis, 2, 1993, 87-110; Lootsma F., Multi-Criteria Decision Analysis via Ratio and Difference Judgement. Kluwer Academic Publishers, Dordrecht, 1999);
- RE – Robust Estimation technique based on the ratio transformation to the shares of preferences (Lipovetsky S. and Conklin W.M., Robust Estimation of Priorities in the AHP, European J. of Operational Research, 137, 2002, 110-122; Lipovetsky S. and Conklin M., AHP Priorities and Markov-Chapman-Kolmogorov Steady-States Probabilities, International J. of the Analytic Hierarchy Process, 7 (2), 2015, 349-363).

The work presents results of comparisons between these four methods of EM, LN, LS, and RE using several characteristics of closeness for the obtained solutions, including:

- pair correlations and cosines between vectors,
- Saaty compatibility index (*S*-compatibility) described in (Saaty T. L., Theory and Applications of the Analytic Network Process: Decision Making with Benefits, Opportunities, Costs, and Risks. Pittsburgh, PA: RWS Publications, 2005; Saaty T.L. and Peniwati K., Group decision-making: Drawing out and reconciling differences. Pittsburgh, PA: RWS Publications, 2007);
- Garuti compatibility index (*G*-compatibility) described in (Garuti A. C., Measuring compatibility (closeness) in weighted environments, Proceedings of the International Symposium on the AHP, Vina del Mar, Chile, August 2– 6, 2007; Garuti A. C. and Salomon V.A.P., Compatibility indices between priority vectors, International J. of the Analytic Hierarchy Process, 4 (2), 2011, 152-160).
- For different sizes and consistency of the matrices of judgement used in the classical AHP literature, the priority vectors are calculated, their compatibility indices estimated, and characteristics of the matrix fit by the vectors are described.
- In general, the explored methods are simple and convenient, and can significantly facilitate practical applications of the AHP for optimum solutions in various problems.

Priority vectors in several estimations. The AHP pairwise priority ratios matrix in general form can be written as follows:

$$A = \begin{pmatrix} 1 & a_{12} & \dots & a_{1n} \\ a_{21} & 1 & \dots & a_{2n} \\ \dots\dots & \dots\dots & \dots\dots & \dots\dots \\ a_{n1} & a_{n2} & \dots & 1 \end{pmatrix}. \qquad (1)$$

It is a Saaty matrix of pairwise judgements among n items, elicited from an expert. Each element a_{ij} shows a quotient of preference of the i-th item over j-th item in their comparison, so we have transposed-reciprocal elements $a_{ij} = a_{ji}^{-1}$. A theoretical Saaty matrix of pair comparisons defines each ij-th element as a ratio of the unknown priorities w_i and w_j:

$$W = \begin{pmatrix} w_1/w_1 & w_1/w_2 & \cdots & w_1/w_n \\ \text{----} & \text{------} & & \text{---} \\ w_n/w_1 & w_n/w_2 & \cdots & w_n/w_n \end{pmatrix} = w * \left(\frac{1}{w}\right)'. \qquad (2)$$

The vector-column w consists of the elements $w_1, w_2, \dots, w_n$, the vector-row $(1/w)'$ contains the reciprocal values $1/w_1, 1/w_2, \dots, 1/w_n$, and the right-hand side of the relation (2) shows the outer product of these two vectors. From (2), the identical relation for the theoretical matrix and vector is: $Ww = nw$.

For the obtained matrix (1) a similar relation can be presented as the eigenproblem:

$$A\alpha = \lambda\alpha \, , \qquad (3)$$

where the first eigenvector a for the maximum eigenvalue λ defines the vector of priorities. It is the eigenvector method EM of the classical AHP.

- Another known way is the Least Squares estimation for priority vector expressed via the following eigenproblem:

$$(AA')\alpha = \lambda^2\alpha \,. \tag{4}$$

The main vector α yields the priority vector in the LS approach.

- Multiplicative, or Logarithmic technique: priority vector as geometric means of the elements in rows of matrix (1):

$$\alpha_i = \sqrt[n]{\prod_{j=1}^{n} a_{ij}} \,. \tag{5}$$

- The obtained AHP priority vectors are also standardized by the total of the elements:

$$\alpha_{i_{normalized}} = \alpha_i / sum(\alpha_i) \,. \tag{6}$$

It is the priority vector estimation in the LN approach.

- Robust estimation (RE): a theoretical matrix of shares is:

$$U = \begin{pmatrix} w_1/(w_1+w_1) & w_1/(w_1+w_2) & \dots & w_1/(w_1+w_n) \\ ----- & ------- & & ---- \\ w_n/(w_n+w_1) & w_n/(w_n+w_2) & \dots & w_n/(w_n+w_n) \end{pmatrix}, \qquad (7)$$

Each element u_{ij} in (7) is defined as *i*-th priority in the sum of *i*-th and *j*-th priorities:

$$u_{ij} = \frac{w_i}{w_i + w_j} = \frac{w_i/w_j}{1 + w_i/w_j} \ . \qquad (8)$$

To estimate the priority vector using the matrix (7) we write identical equalities:

$$\begin{cases} \frac{w_1}{w_1+w_1}(w_1+w_1) + \frac{w_1}{w_1+w_2}(w_1+w_2) + \ldots + \frac{w_1}{w_1+w_n}(w_1+w_n) = nw_1 \\ - \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad - \\ \frac{w_n}{w_n+w_1}(w_n+w_1) + \frac{w_n}{w_n+w_2}(w_n+w_2) + \ldots + \frac{w_n}{w_n+w_n}(w_n+w_n) = nw_n \end{cases} . \qquad (9)$$

Then with notation (8) we present the system (9) as following:

$$\begin{cases} (u_{11} + \sum_{j=1}^{n} u_{1j})w_1 + u_{12}w_2 + \ldots + u_{1n}w_n = nw_1 \\ - \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad - \\ u_{n1}w_1 + u_{n2}w_2 + \ldots + (u_{nn} + \sum_{j=1}^{n} u_{nj})w_n = nw_n \end{cases} . \qquad (10)$$

or in the matrix form $\quad \big(U + diag(Ue)\big)w = nw$

Again, In the matrix form the system (10) can be written as:

$$\big(U + diag(Ue)\big)w = nw, \tag{11}$$

where U is the matrix (7), e denotes a uniform vector of n-th order, and $diag(Ue)$ is a diagonal matrix of totals in each row of matrix U.

In the AHP, the pair ratios w_i/w_j (2) are estimated by elicited values a_{ij} (1). Using a_{ij} in (8), we obtain the empirical estimates b_{ij} of the pairs' shares:

$$b_{ij} = \frac{a_{ij}}{1+a_{ij}} \,. \tag{12}$$

This transformation for matrix A (1) yields a pairwise share matrix B with the elements (12). These elements (12) are positive, less than one, and have a property $b_{ij} + b_{ji} = 1$. The transposed elements b_{ij} and b_{ji} are skew-symmetrical off the diagonal b_{ii}=0.5, so $b_{ij} - b_{ii} = -(b_{ji} - b_{ii})$.

- For empirical Saaty matrix A (1) we have the eigenproblem (3) in place of the theoretical relations (2). Similarly, using the empirical skew-symmetric matrix B (12) in place of theoretical matrix U, we represent the system (11) as the eigenproblem:

$$(B + diag(Be))\alpha = \lambda\alpha, \tag{13}$$

where α as the main eigenvector. It is the RE - robust estimation of the priority vector.

- **Measures of Consistency, Compatibility, and Precision**

Due to the general methodology of AHP, the so-called consistency index (CI) equals

$$CI = \frac{\lambda - n}{n-1} \tag{14}$$

where λ is the maximum eigenvalue of the matrix in the problem (3), and n is the matrix order. The so-called random consistency index (RI) is a constant tabulated in the AHP for various n, and the consistency ratio (CR) equals:

$$CR = \frac{CI}{RI}. \tag{15}$$

A value CR up to 10% is considered as indicating a small inconsistency in the matrix of pairwise comparisons (1), so such a matrix is acceptable, otherwise, with CR>10%, the data could require a reviewing of the elicited judgements.

- For comparisons between the obtained solutions, several characteristics can be applied. Among those are the pairwise correlation between the elements of two vectors, which can be reduced to the expression:

$$r(x,y) = \frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i-\bar{x})^2 \sum_{i=1}^{n}(y_i-\bar{y})^2}} = \frac{\sum_{i=1}^{n} x_i y_i - \frac{1}{n}}{\sqrt{\sum_{i=1}^{n} x_i{}^2 - \frac{1}{n}}\sqrt{\sum_{i=1}^{n} y_i{}^2 - \frac{1}{n}}}, \tag{16}$$

where a bar above the variables denotes the mean values. Without the items $1/n$ for centering the measure (16) coincides with the cosine as normalized projection of one vector onto another one.

- Another good measure of closeness between two vectors is the so-called Saaty compatibility index (*S*-compatibility) described in (Saaty, 2005; Saaty and Peniwati, 2007; Garuti and Salomon, 2011). This index can be found as follows.

For two vectors x and y of an n-th order, build a matrix X with elements defined as quotients $X_{ij}=x_i/x_j$ of the components x, and a matrix Y with its elements defined as quotients $Y_{ij}=y_i/y_j$ of the components of y. Take the transposed matrix Y' with the elements $Y'_{ij}=y_j/y_i$ and find the Hadamard element-wise product of these two matrices $X*Y'$.

The *S*-index is defined as the normalized total of the elements of this matrix:

$$S=\frac{1}{n^2}\sum_{i,j=1}^{n}X_{ij}Y'_{ij}=\frac{1}{n^2}\sum_{i,j=1}^{n}\frac{x_i}{x_j}\frac{y_j}{y_i}. \qquad (17)$$

If two vectors coincide, this index equals 1. Within 10% of discrepancy, when $S\leq 1.1$, the vectors are considered as compatible; otherwise, when $S>1.1$, they are considered as incompatible.

- A further development of a compatibility measure in the compatibility index *G* was given in (Garuti, 2007; Garuti and Salomon, 2011):

$$G=\sum_{i=1}^{n}\frac{min(x_i,y_i)}{max(x_i,y_i)}\frac{x_i+y_i}{2}. \qquad (18)$$

Closer to 1 values, with $G > 0.9$, correspond to the compatible vectors; the values $G < 0.9$ correspond to incompatible vectors.

- To check precision of fit for the pairwise judgements by the priority estimate, we can use definition of the elements a_{jk} as quotients of preference between each pair of j-th and k-th items. With a vector-column of priority estimate α, find its element-reciprocal vector-row $(1/\alpha)'$ and build their outer product by the same pattern as used in (2). With this outer product find quality of fit for the matrix A (1). The standard error (STE) is a measure of the mean distance between the observed and estimated pairwise ratios:

$$STE = \sqrt{\frac{1}{n^2}\sum_{j,k=1}^{n}\left(a_{jk} - \frac{\alpha_j}{\alpha_k}\right)^2} \quad . \qquad (19)$$

- Another measure of the precision for matrix approximation by vector is the Mean Absolute Error (MAE):

$$MAE = \frac{1}{n^2}\sum_{j,k=1}^{n}\left|a_{jk} - \frac{\alpha_j}{\alpha_k}\right| \quad . \qquad (20)$$

The smaller values of fit correspond to better quality of a vector estimate.

- Another convenient measure is an analogue of the coefficient of multiple determination R^2 widely used in regression analysis:

$$R^2 = 1 - \frac{RSS}{ESS} = 1 - \frac{\sum_{j,k=1}^{n}\left(a_{jk} - \frac{\alpha_j}{\alpha_k}\right)^2}{\sum_{j,k=1}^{n}\left(a_{jk} - 1\right)^2} . \qquad (21)$$

In the numerator is the residual sum of squares (*RSS*) of the estimated priority deviations from the elicited values, in the denominator is the equivalent sum of squares (*ESS*) which assumes all the same preferences $\alpha_j / \alpha_k \equiv 1$. The coefficient (21) shows how much the found priorities outperform the case of absence of preferences among the alternatives.

The better is approximation of the paired judgements by the estimated priorities – the closer is *RSS* to zero, so the coefficient of determination R^2 is bigger and closer to one. In absence of preferences $\alpha_j/\alpha_k = 1$, the numerator equals the denominator, and $R^2 = 0$. For the exact fit $a_{jk} = \alpha_j/\alpha_k$ for all judgements, $RSS = 0$, and $R^2 = 1$.

- The value R^2 commonly belongs to the interval from 0 to 1, that makes it a very convenient measure for comparison of the priority vectors obtained by different techniques. Only really poor estimates can produce the residual total *RSS* above the value of the equivalent residuals *ESS*, and it would be indicated by the negative R^2. The characteristic (21) corresponds to *STE* measure (19) of squared deviations, but it is possible to build the other estimates, for example, using the MAE residuals (20) as well.

Numerical comparisons for priority estimations. Example 1: Consider the problem of "Choosing the best home", described in (Saaty and Vargas, 1994; Saaty, 1996). The criteria are: 1 – size of house, 2 – location to bus, 3 – neighborhood, 4 – age of house, 5 - yard space, 6 – modern facilities, 7 – general condition, 8 – financing. The matrix of pairwise comparisons A (1) for this problem is presented in Table 1a. The consistency index and consistency ratio are: $CI = \frac{9.669-8}{7} = 0.238$, $CR = \frac{0.238}{1.41} = 0.169$. This value of CR indicates some mild inconsistency.

Table 1a. Example 1: Choosing the best home problem. Pairwise comparison matrix.

item	1	2	3	4	5	6	7	8
1	1	5	3	7	6	6	1/3	1/4
2	1/5	1	1/3	5	3	3	1/5	1/7
3	1/3	3	1	6	3	4	6	1/5
4	1/7	1/5	1/6	1	1/3	1/4	1/7	1/8
5	1/6	1/3	1/3	3	1	1/2	1/5	1/6
6	1/6	1/3	1/4	4	2	1	1/5	1/6
7	3	5	1/6	7	5	5	1	1/2
8	4	7	5	8	6	6	2	1

- **Table 1b.** Example 1: Choosing the best home problem. Priority vector estimations.

item	EM	LS	LN	RE
1. size of house	0.173	0.199	0.175	0.150
2. location to bus	0.054	0.100	0.063	0.054
3. neighborhood	0.188	0.148	0.149	0.141
4. age of house	0.018	0.017	0.019	0.022
5. yard space	0.031	0.045	0.036	0.037
6. modern facilities	0.036	0.065	0.042	0.041
7. general condition	0.167	0.184	0.167	0.163
8. financing	0.333	0.242	0.350	0.392
Correlations				
EM	1	0.935	0.988	0.972
LS	0.935	1	0.933	0.881
LN	0.988	0.933	1	0.991
RE	0.972	0.881	0.991	1
S-compatibility				
EM	1	1.113	1.015	1.028
LS	1.113	1	1.071	1.122
LN	1.015	1.071	1	1.010
RE	1.028	1.122	1.010	1
G-compatibility				
EM	1	0.774	0.927	0.865
LS	0.774	1	0.809	0.742
LN	0.927	0.809	1	0.912
RE	0.865	0.742	0.912	1
Precision				
STE	2.071	1.849	1.813	1.831
MAE	1.079	1.083	0.958	0.934
R^2	0.423	0.540	0.558	0.549

- For comparison between the four priority vectors the measures of Correlations, S- and G- compatibility are given in three matrices in Table 1b.
- Judging by correlations, all the vectors are close enough by their structure, and LS is a bit further from the three others.
- The *S*-compatibility proves that the EM, LN, and RE vectors are similar, within less than the required threshold of 10% of *S*-index deviation from one.
- The more sensitive *G*-compatibility demonstrates that the pair of EM and LN vectors are close with G=0.927, and two vectors LN and RE are close with G=0.912, which are the values above the threshold 0.9 needed for the vectors to be seen as compatible.
- The last part at the bottom of Table 1b displays the precision for each vector solution.
- By the minimum standard error *STE* – the best model is LN, and by the mean absolute error MAE – the best model is RE. The values of MAE suggest also that an average deviation from the observed pair judgements evaluated by the obtained quotients from a priority vector is not more than one unit.
- The coefficient of multiple determination R^2 in the last row of Table 1b shows by its maximum values that the models LN and RE outperform the other two models, though all R^2 values are not high, which indicates a difficulty in approximation of inconsistent judgements by a priority vector in any estimation.

- **Example 2**: the problem of "Distance from Philadelphia" is one of the first AHP problems described by Saaty (1977). The remoteness of six cities from Philadelphia was estimated by the criterion: for each pair of cities, how many times farther the more distant city is located from Philadelphia than the nearer one is? The elicited data is presented in Table 2a.

- **Table 2a.** Example 2: Distance from Philadelphia problem. Pairwise comparison matrix.

Airport	CAI	TYO	ORD	SFO	LGW	YMX
Cairo.CAI	1	0.333	8	3	3	7
Tokyo.TYO	3	1	9	3	3	9
Chicago.ORD	0.125	0.111	1	0.167	0.2	2
SanFrancisco.SFO	0.333	0.333	6	1	0.333	6
London.LGW	0.333	0.333	5	3	1	6
Montreal.YMX	0.143	0.111	0.5	0.167	0.167	1

- The maximum eigenvalue in this example equals $\lambda = 6.454$. The consistency index and consistency ratio are:

- $$CI = \frac{6.454-6}{5} = 0.091, \quad CR = \frac{0.091}{1.24} = 0.073\ .$$

- The value *CR*=7.3% lesser than 10% permits to conclude that the data on pair judgements is sufficiently consistent.

- **Table 2b.** Example 2: Distance from Philadelphia problem. Priority vector estimations.
- Besides four estimations, here is the additional vector of the actual shares of distances known in this case.

City	EM	LS	LN	RE	actual
1. Cairo	0.262	0.254	0.260	0.239	0.278
2. Tokyo	0.397	0.305	0.399	0.447	0.361
3. Chicago	0.033	0.047	0.035	0.034	0.032
4. San Francisco	0.116	0.186	0.116	0.104	0.132
5. London	0.164	0.184	0.163	0.147	0.177
6. Montreal	0.027	0.024	0.027	0.029	0.019
Correlations					
EM	1	0.943	1.000	0.990	0.991
LS	0.943	1	0.941	0.898	0.973
LN	1.000	0.941	1	0.991	0.990
RE	0.990	0.898	0.991	1	0.962
actual	0.991	0.973	0.990	0.962	1
S-compatibility					
EM	1	1.064	1.000	1.009	1.024
LS	1.064	1	1.064	1.106	1.045
LN	1.000	1.064	1	1.008	1.027
RE	1.009	1.106	1.008	1	1.060
actual	1.024	1.045	1.027	1.060	1
G-compatibility					
EM	1	0.821	0.993	0.900	0.914
LS	0.821	1	0.820	0.753	0.854
LN	0.993	0.820	1	0.905	0.908
RE	0.900	0.753	0.905	1	0.823
actual	0.914	0.854	0.908	0.823	1
Precision					
STE	1.390	1.340	1.333	1.523	2.295
MAE	0.696	0.862	0.686	0.790	1.012
R^2	0.794	0.809	0.810	0.753	0.438

- We see that in general the vectors are similar and each one makes sense as proportionally scaled distances from Philadelphia to other cities in the USA, as well as to other countries and continents.
- The pair correlations also show that the vectors are closely related to the actual distances, and the same is supported by *S*-compatibility index.
- *G*-compatibility indicates that EM and LN vectors are compatible with the actual shares of distances.
- The precision of the reproduction of the judgement matrix is high, especially by the LS and LN methods.
- Precision measured by *STE*, *MAE*, and R^2 of the actual distances is the worst one within the other values in the last rows of Table 2b.
- It means that the pair judgements on distances correspond rather to the priority vectors than to the actual distance shares.
- Thus, in this data we do not need to use the actual data in considering compatibility among the estimated vectors.

- **Example 3.** The data for this problem is given in (Whitaker R., Validation examples of the Analytic Hierarchy Process and Analytic Network Process. Mathematical and Computer Modelling, 46, 2007, 840–859) where the area of five geometric figures were compared – see the matrix of pair judgements in Table 3a.
- **Table 3a.** Example 3: Geometric figures' area problem. Pairwise comparison matrix.

figure	Circle	Triangle	Square	Diamond	Rectangle
Circle	1	9	2.5	3	6
Triangle	0.111	1	0.2	0.286	0.667
Square	0.4	5	1	1.7	3
Diamond	0.333	3.5	0.588	1	1.5
Rectangle	0.167	1.5	0.333	0.667	1

- The maximum eigenvalue of this matrix is $\lambda = 5.026$. The random consistency for *n*=5 is *RI*=1.12, so consistency index and consistency ratio equal the following values:

$$CI = \frac{5.026-5}{4} = 0.006, \quad CR = \frac{0.006}{1.12} = 0.006\ .$$

The *CR*=0.6% proves a very high level of consistency of this data. It can be explained by the used pairwise ratios where not only the integer numbers but also the rational numbers (like 2.5 or 3.5) were permitted in the preference evaluation.

- **Table 3b.** Example 3: Geometric figures' area problem. Priority vector estimations, with additional vector of the actual shares of the areas measured for these figures. All the vectors look practically coinciding, the pair correlations are very high, and both *S*- and *G*- indices prove compatibility among the estimates and with the actual observations. The precision measured by *STE*, *MAE*, and R^2 characteristics demonstrates a high quality of the data fit by any of the estimated vectors of priority and by the actual values as well.

figures	EM	LS	LN	RE	actual
1. Circle	0.488	0.464	0.487	0.496	0.470
2. Triangle	0.049	0.050	0.049	0.049	0.050
3. Square	0.233	0.248	0.233	0.225	0.240
4. Diamond	0.148	0.159	0.148	0.147	0.140
5. Rectangle	0.082	0.078	0.082	0.083	0.090
Correlations					
EM	1	0.998	0.999	0.999	0.999
LS	0.998	1	0.998	0.995	0.998
LN	0.999	0.998	1	0.999	0.999
RE	0.999	0.995	0.999	1	0.998
actual	0.999	0.998	0.999	0.998	1
S-compatibility					
EM	1	1.003	1.000	1.000	1.003
LS	1.003	1	1.003	1.004	1.007
LN	1.000	1.003	1	1.000	1.003
RE	1.000	1.004	1.000	1	1.003
actual	1.003	1.007	1.003	1.003	1
G-compatibility					
EM	1	0.948	0.999	0.982	0.955
LS	0.948	1	0.948	0.931	0.953
LN	0.999	0.948	1	0.982	0.956
RE	0.982	0.931	0.982	1	0.940
actual	0.955	0.953	0.956	0.940	1
Precision					
STE	0.253	0.195	0.253	0.289	0.279
MAE	0.145	0.110	0.145	0.162	0.165
R^2	0.987	0.992	0.987	0.983	0.985

- **Conclusions:** The paper considered several methods of priority vector evaluation in the AHP: the classical eigenproblem method, least squares, multiplicative or logarithmic approach, and a robust estimation based on transformation of the pairwise ratios to the shares of preferences. Together with estimation of the vectors, validation of data consistency and comparison of vectors by correlations, *S*- and *G*- compatibility indices were completed.
- Numerical results for different data sizes and consistency indices demonstrate that all the methods produce compatible results for the consistent data, otherwise a discrepancy between different methods of the priority estimation could be observed. Therefore, the data consistency should be always proved before the vector evaluation.
- Another important point – the precision assessment of the data matrix approximation by the obtained priority vectors. Any regular statistical modeling requires a verification of the produced results by some quality characteristics. For example, in regression analysis, such measures as the residual standard error *STE*, mean absolute error *MAE*, and coefficient of multiple determination R^2 are commonly employed. Applying them in the AHP environment can enrich the evaluation and interpretation of the results on priority modeling.
- Example 1 with a low consistency, shows that R^2 values are also not high which indicates a difficulty of approximation of inconsistent judgements by a priority vector in any estimation; and by *MAE* values a mean deviation of the quotients of a priority vector's elements from the observed pair judgements could reach one. Example 2 with a good consistency, yields the precision of the reproduction of the judgement matrix by the found priority vectors is high enough, although at the same time the actual distances occurred to show the worst vector for approximation of the elicited pairwise priority matrix. Example 3 with a perfect consistency yields all vectors of high compatibility and quality of the elicited judgements produced by each vector's quotients of preference.

- Resuming, the considered methods of priority vector estimation and characteristics of their quality are convenient and helpful in practical applications of the AHP for solving various multiple-criteria decision making problems.

Thanks!