# COMPARISON SUPPORT SYSTEM BASED ON COMPARISON PATTERN SEARCH FOR ANALYTIC HIERARCHY PROCESS

Yumi Tadano*
Graduate School of Information Science and Technology
Hokkaido University
Sapporo, Hokkaido, Japan
E-mail: yumi-hr@complex.eng.hokudai.ac.jp

Hidenori Kawamura
Graduate School of Information Science and Technology
Hokkaido University
Sapporo, Hokkaido, Japan
E-mail: kawamura@complex.eng.hokudai.ac.jp

Keiji Suzuki
Graduate School of Information Science and Technology
Hokkaido University
Sapporo, Hokkaido, Japan
E-mail: suzuki@complex.eng.hokudai.ac.jp

**ABSTRACT**

The analytic hierarchy process (AHP) is a method for decision making that considers uncertain situations or multiple evaluation criteria. In the AHP, a decision maker compares two elements between evaluation criteria and alternatives. Therefore, comparing all pairs is difficult when evaluating many alternatives. In this study, we present the "comparison support method" for evaluating many alternatives when a decision maker needs to decide the highest priority alternative. The comparison support method stops pairwise comparisons when the best solution, i.e., the highest priority alternative, is found, even if all pairs have not been compared. We represented a modeling of determine the best alternative using the AHP include comparison support system and an application to determine the best TV.

Keywords: Analytic Hierarchy Process, Decision Modeling and Theory, Information Technology

## 1. Introduction

The analytic hierarchy process (AHP) (Saaty, 1994, 1990, 2004) is a decision making support method for expressing human subjective judgments numerically. The AHP calculates overall evaluations by structuring a hierarchy of the problem and comparing pairs of elements at each level. Several decision making problems are solved by using the AHP (Azis, 1990, Tummala, Chin, and Ho, 1996, Vaidya, and Kumar, 2006) because human subjective judgments including preference or guess can be expressed numerically by the pairwise comparisons. In addition, the AHP is straightforward. In the AHP, a decision maker inputs values of pairwise comparisons between evaluation criteria and alternatives. Comparing all pairs becomes difficult when evaluating many alternatives. We present the "comparison support method" to address these difficulties by terminating pairwise comparisons when the best solution is found even if all comparisons have not been finished.

* Corresponding author

## 2. Amount of time for pairwise comparison in AHP

The AHP calculate priorities after decision maker inputs values of all paired comparisons between the evaluation criteria and alternatives. This is called the relative measurement method. In this method, for *m* evaluation criteria and *n* elements, the amount of time for pairwise comparison is represented by the following equation.

$$ {}_mC_2 + {}_nC_2 \times m = m(m-1)/2 + nm(n-1)/2 \qquad (1) $$

## 3. The comparison support method

When a decision maker calculates priorities using the AHP, there are several conditions: the relative priorities of each alternative are needed; the alternative that has the highest priority is needed, etc. The AHP even compares low priority alternatives. The pairwise comparisons that do not influence the decision about the highest priority alternatives are included in the comparisons between low priority alternatives. Therefore, the pairwise comparisons include omissible comparisons when a decision maker needs the alternative that has the highest priority.

We presented the "comparison support method" (Tadano, Kawamura, Suzuki, and Ohuchi, 2010) for evaluating many alternatives when a decision maker needs to decide the highest priority alternative. The comparison support method terminates pairwise comparisons when the best solution, i.e., the highest priority alternative, is found, even if all pairs have not been compared.

## 4. Modeling of determine the best alternative using the AHP include the comparison support system

This section defines determination of the best alternative using the AHP include comparison support system when a three-level hierarchy is assumed.

### 4.1 Definitions

(a) Evaluation Criterion Set $E$

Let $E=\{e_1, e_2, \ldots, e_m\}$ be an evaluation criterion set that consist of *m* criteria.

(b) Alternative Set $A$

Let $A=\{a_1, a_2, \ldots, a_n\}$ be an alternative set that consist of *n* alternative.

(c) Alternative Function $c$

$c_{ea}$ represents a function of alternative $a$ for evaluation criterion $e$.

(d) Importance Intensity Set $S$

Let $S=\{1/t, 1/t\text{-}1, \ldots, 1, 1, 2, \ldots, t\}$ be an importance intensity set that represents a scale of absolute numbers used to assign numerical value to judgments made by comparing to elements. Then $t$ is an positive integer.

(d) Weight of Evaluation Criterion $W^0$

The weight $w^0_e$ denotes the relative importance of evaluation criterion $e$. It is divided into following three cases based of whether criterion $e$ is an absolute requirement or unnecessary requirement or others. The absolute requirement is the requirement that must be satisfied. If the absolute requirement is not satisfied in one alternative, it is never chosen for the best alternative. On the other hand, in the unnecessary requirement, any function is not cared for the decision maker.

i) Criterion $e$ is an absolute requirement when
$w^0_e = \infty$

ii) Criterion $e$ is an unnecessary requirement when
$w^0_e = 0$

iii) Criterion $e$ is the others when
Calculate following pairwise comparison matrix between evaluation criteria $X^0$

(e) Pairwise comparison matrix between evaluation criteria $X^0$

Let $X^0=(x^0_{ij})$ be a pairwise comparison matrix between evaluation criteria. $x^0_{ij}$ represents a value of pairwise comparison that compare $e_i$ with $e_j$. Hence, $x^0_{ij} \in S$ and the larger the value of $x^0_{ij}$, $e_i$ is the more importance than $e_j$. Then $x^0_{ii}=1$, $x^0_{ij}=1/x^0_{ji}$.

(f) Weight of Alternative $W^e$

The weight $w^e_a$ denotes the relative importance of the alternative $a$ about the evaluation criterion $e$. It is divided into following two cases based of whether the alternative function $c_{ea}$ is dependently or independently from the structural effects.

i) Alternative function $c_{ea}$ is a dependently from the structural effects when
Calculate using for absolute measurement method (Saaty, 1986) and so on

ii) Alternative function $c_{ea}$ is independently when
Calculate following pairwise comparison matrixes between alternatives $X^e$

(g) Pairwise comparison matrix between alternatives $X^e$

Let $X^e=(x^e_{ij})$ be a pairwise comparison matrix between alternatives about the evaluation criteria $e$. $x^e_{ij}$ represents a value of pairwise comparison that compare $c_{ei}$ with $c_{ej}$. Hence, $x^e_{ij} \in S$ and the larger the value of $x^e_{ij}$, $c_{ei}$ is the more importance than $c_{ej}$ Then $x^e_{ii}=1$, $x^e_{ij}=1/x^e_{ji}$.

(h) Final weight $W$

Let $w_a$ be the final weight of alternative $a$. It is divided into following two cases based of whether alternative function $c_{ea}$ that evaluation criteria $e$ is an absolute requirement, satisfied its requirement or not.

i) Satisfied the requirement of the alternative function $c_{ea}$ that the evaluation criteria $e$ is an absolute requirement when

$$w_a = \sum_{e=1}^{m} w_e^0 \cdot w_a^e$$

ii) Not satisfied the requirement when

$$w_a=0$$

(i) The Best Alternative $a_{best}$

Let $a_{best}$ be the best alternative that $w_a \geqq w_i$ $(\forall w_i)$

(j) Set of Pairwse Comparison Value $X'$

Let $X'$ be a set of pairwise comparison values at a point in time. hence, $X'=\{(x, e, i, j): x\in(*\cup S), (e\in E, i\in A, j\in A)\cup(e=0, i\in E, j\in E)\}$. Then, $i < j$, $|X'| = m(m-1)/2+m*n(n-1)/2$. "*" represents the case that value has not been decided yet. Let $r$ be a number of X' elements that $x\neq *$. Hence, $r$ represents the numbers of pairwise comparison that decision maker decide at a point in time.

(k) Set of Possible Pairwise Comparison Value's Set $Y$

Let $Y$ be a set of possible comparison value's set about the elements that $x=*$ in pairwise comparison value's set $X'$. $|Y|=S^k$ then k be a number of element that $x=*$.

### 4.2 Formulation

We replace the determination of the best alternative with a problem that minimizes the number of pairwise comparison $r$ as an objective function when evaluation criteria $E$ and alternative $A$ are given.

Constraints represent that in all evaluation criteria $E$ and alternatives $A$ alternative function $c_{ea}$ are exist and in all set of possible pairwise comparison value's set $Y$ the best alternative $a_{best}$ are the same.

## 5. Algorithm

The following is the algorithm when determine the best alternative using the AHP include the comparison support system.

step1. Determine evaluation criteria $E$, alternative $A$, importance intensity set $S$

step2. Determine each evaluation criteria are absolute requirement or unnecessary requirement or others.

step3. Determine weights of alternative $w^e_a$ that evaluation criterion $e$ is an unnecessary requirement and alternative function $c_{ea}$ is dependently from the structural effects.

step4. Make pairwise comparison matrixes about the evaluation criteria and the alternatives.

step5. Compare two evaluation criteria or alternatives about certain evaluation criteria and input a value in the pairwise comparison matrix's element.

step6. Enumerate set of possible pairwise comparison value's set $Y$ and each best alternative $a_{bast}$. Investigate all $a_{best}$ are the same or not. When $a_{best}$ that different from others is found, return to step5. If all $a_{best}$ are the same, outputs the $a_{best}$ and terminate pairwise comparison.

## 6. Application to determine the best TV

In this section we represent the application of the AHP include comparison support system to determine the best TV. Evaluation criteria $E$, alternative $A$, importance intensity set $S$ represent followings.

- $E$={Brand, Screen size, Dynamic contrast ratio, 3D, Number of tuner, Internet, Brightness sensor, Movement sensor, USB HD, Built in HD, Built in BD recorder, Cost, Design}. Then, "Brand" and "Design" are independently from the structural effects.
- $A$={TV1, TV2, TV3, TV4, TV5, TV6, TV7}
- $S$={1/3, 1/2, 1, 2, 3}

Figure1 represents the hierarchy of determination the best TV. Table1 represents each alternative function $c_{ea}$. In the evaluation criteria, absolute requirements are {USB HD＝ok, Cost＜100000}, unnecessary requirements are {Brand, Dynamic contrast ratio, Built in HD} and others are represented at Table2. Then, the weights of the evaluation criteria those not absolute and unnecessary requirements are determined using the absolute measurement method and so on. Table3 and 4 represent an example of pairwise comparison value and comparison order. In each element, the number in parentheses represents the comparison order and the gray cells represent that the pairwise comparison have not been yet. In this example, we considered the 38 pairs at the upper right of the each pairwise comparison matrixes. It is clear that using the comparison support system decrease the number of pairwise comparison to 25 and the best alternative is TV1. The best alternative is calculated from 66% of all pairwise comparisons.

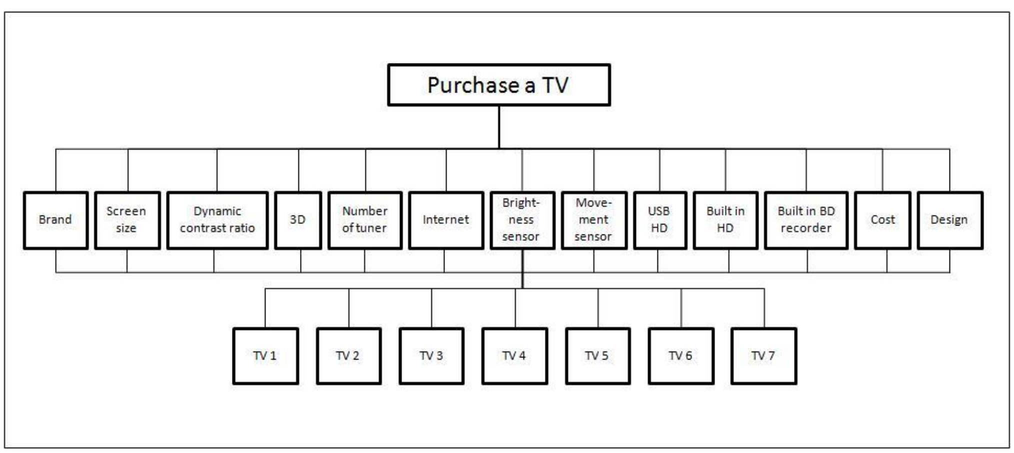


Figure1. Hierarchy for Purchase a TV

Table1. Each alternative function of TV

| | TV1 | TV2 | TV3 | TV4 | TV5 | TV6 | TV7 |
|---|---|---|---|---|---|---|---|
| Brand | SHARP DZ3 | SHARP DZ3 | SHARP LB3 | TOSHIBA X2 | TOSHIBA ZG1 | TOSHIBA RE1 | TOSHIBA A1 |
| Screen size | 20v | 20v | 46v | 55v | 42v | 32v | 22v |
| Dynamic contrast ratio | 2000000:1 | 2000000:1 | 5000000:1 | 9000000:1 | 2000000:1 | 2000000:1 | 50000:1 |
| 3D | no | no | ok | ok | ok | no | no |
| Number of tuner | 2 | 2 | 2 | 3 | 3 | 2 | 1 |
| Internet | ok | ok | ok | ok | no | ok | no |
| Britghtness sencor | ok | ok | ok | ok | ok | ok | ok |
| Movement sensor | ok | ok | no | no | no | no | no |
| USB HD | ok | ok | ok | ok | ok | ok | no |
| Built in HD | no | no | no | ok(3TB) | no | no | no |
| Built in BD recoder | no | no | ok | no | no | no | no |
| Cost(¥) | 50000 | 45000 | 356400 | 685000 | 145374 | 63800 | 34980 |
| Design |  |  |  |  |  |  |  |

Table2. Weights of evaluation criteria those not absolute and unnecessary requirements

| Evaluation criteria | $W^e$ |
|---|---|
| Screen size | 20v: *w*=0.1, 22v: *w*=0.2, 32v: *w*=0.5, 46v: *w*=0.15, 55v: *w*=0.05 |
| 3D | ok: *w*=1, no: *w*=0 |
| Number of tuner | 1: *w*=0.1, 2: *w*=0.4, 3: *w*=0.5 |
| Internet | ok: *w*=1, no: *w*=0 |
| Brightness sensor | ok: *w*=1, no: *w*=0 |
| Movement sensor | ok: *w*=1, no: *w*=0 |
| Built in BD recorder | ok: *w*=1, no: *w*=0 |

Table3. Pairwise comparison value and comparison order between evaluation criteria

| | Screen size | 3D | Number of tuner | Internet | Brightness sensor | Movement sensor | Built in BD recoder | Design |
|---|---|---|---|---|---|---|---|---|
| Screen size | | 1/3(13) | 1/2(17) | 1/3(12) | 1(23) | 1/3(24) | 1/3(14) | 1(11) |
| 3D | | | 1(16) | | 2(1) | 1/2(2) | 1/2(15) | 2(3) |
| Number of tuner | | | | 1/2(30) | 2(20) | 1/2(21) | 1/2(6) | 2(22) |
| Internet | | | | | 3(18) | 1(19) | | |
| Britghtness sencor | | | | | | | 1/3(4) | |
| Movement sensor | | | | | | | 1(5) | |
| Built in BD recoder | | | | | | | | 3(7) |
| Design | | | | | | | | |

Table4. Pairwise comparison value and comparison order between alternatives

| | TV1 | TV2 | TV3 | TV4 | TV6 |
|---|---|---|---|---|---|
| TV1 | | 2(8) | 3(9) | 3(10) | 3(25) |
| TV2 | | | | | |
| TV3 | | | | | |
| TV4 | | | | | |
| TV6 | | | | | |

## 8. Discussion

It is clear that using the AHP include the comparison support system decrease the number of pairwise comparison. The comparison support system can apply to other AHP's method because it is independently from the AHP's method.

## 9. Conclusion

In this paper, we focused on a case where the decision maker needs the highest priority alternative. We presented a comparison support method for solving the difficulty of comparing all pairs when evaluating many alternatives. The comparison support method terminates the pairwise comparisons when the best solution is found, even if all pairs have not been compared. We represented a modeling of determine the best alternative using the AHP include the comparison support system and an application to determine the best TV.

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