THE CENTROID EXTENSION TO AHP

David L. Olson

Department of Business Analysis & Research

Texas A&M University 77843

(409) 845-2254

The analytic hierarchy process has proven highly useful in selecting alternatives with multiple evaluation criteria. AHP has also been used to support decisions through mathematical programming, and could be used to support evaluation of many alternatives, following the concept of expert systems. These applications would require development of a set of weights on criteria. Analytic hierarchy process uses ratio estimates from pairwise comparisons. AHP absolute measurement uses ratio estimates of alternative class performance on each criteria, also from pairwise comparisons. The centroid of weights is presented as an alternative means of weighting hierarchical elements, using only ordinal information. Formulation for the centroid of feasible weight values is presented with a table of values for cases where the preference ranking includes no ties. The centroid approach is compared with pairwise comparisons using absolute measurement. The pairwise comparison approach is expected to give decision makers more ability to fine tune weights. The centroid approach is expected to be more robust, and to require much less input with little loss of accuracy.

THE CENTROID EXTENSION TO AHP

INTRODUCTION

The analytic hierarchy process (Saaty [1977, 1988a, 1988b]) is a well developed, highly useful means of aiding choice among a finite set of alternatives. AHP has been applied in many decision contexts (Zahedi [1986]; Shim [1989]). With AHP, the relative ratio importance of criteria and subcriteria are developed through decision maker pairwise comparison, followed by pairwise comparison of alternative performance on each of the subcriteria or criteria.

A number of techniques have been developed for obtaining sets of weights for combining multiobjective functions. Such weights have been useful in mathematical programming, as starting information in ELECTRE (Roy [1978]), and as the basis for expert systems capable of evaluating large numbers of alternatives. At least three studies utilizing AHP in MOLP have been published (Bard [1986]; Mitchell and Bingham [1986]; Olson, et al. [1986]). In applications for selection among a large number of alternatives, weights are usually used as a means for filtering a long list of alternatives down to a shorter list for more detailed consideration by decision makers. Approaches for developing weights include multiattribute utility theory (MAUT - Keeney and Raiffa [1976], regression (Krovak [1987]), and simple weighting and ranking techniques (Schoemaker and Waid [1982]; Belton [1986]). Each of these techniques requires varying levels of input from decision makers, but the intent of all of them is to provide a means of selecting among a set of alternative potential decisions X_j {j alternatives} while reflecting multiple decision objectives O_k {k objectives}. The assumption is generally made that for each alternative X_j, a measure of value V_{jk} can be obtained (objectively or subjectively) for each objective O_k. With the exception of MAUT (which can adopt a nonlinear estimate of utility), these methods share a resulting additive value function of the form:

$$\text{Value}(X_j) = \sum_{i=1}^{k} W_k V_{jk} X_j \quad \text{for } j = 1,...,J$$

Usually, $\sum_{i=1}^{k} W_k = 1$, which can easily be accomplished by normalization. The weights, W_k, can be viewed as the relative importance of each objective k. Note that when used as weights, the measures of value V_{jk} should be scaled to a common metric, such as a maximum of 1 for an ideal measure, and a minimum of 0 for a totally unacceptable measure, in order not to dilute the relative importance provided by the weights W_k. Belton [1986] discussed the differences in AHP and MAUT (multi-attribute value function in Belton's terminology) for eliciting weights.

Each of the techniques to obtain W_k have varying amounts of decision maker input. MAUT can involve a fairly extensive examination of tradeoffs between objectives. AHP relies upon subjective pairwise comparison of hierarchy elements. A regression approach would require a subjective assessment of overall value for each sample alternative X_j. The weighting and rating approach would require less input, as decision makers would simply subjectively assign the W_k. In general, one would expect that the more effort that was devoted to the approximation of W_k, the more accurate the resulting weights.

ANALYTIC HIERARCHY PROCESS

AHP provides a means to convert subjective assessments into a scalar value reflecting the ratio of relative attainments of each alternative to the set of criteria included in the hierarchy. Three steps are involved: (1) problem decomposition; (2) comparative judgement; and (3) synthesis. Problems are decomposed, yielding a hierarchy of objective factors. The intent is for the decision maker to develop a collectively exhaustive list of objective factors bearing upon a decision. Because of limitations of concentration, these factors are arranged in a hierarchy of elements and subelements. Saaty [1977] recommended no more than seven subelements for consideration at one time. Subjective assessment is accomplished by subjective pairwise comparison at each node of the hierarchy. Saaty recommended use of the eigen vector of each pairwise comparison matrix in order to gain a consistent estimate of relative weights. In AHP, alternative decisions comprise the bottom level of the hierarchy, continuing the subjective comparison of each alternative with respect to each objective factor.

AHP for few alternatives

Develop Hierarchy	eigenvector of pairwise comparisons
Alternatives ≤ 7	eigenvector of pairwise comparisons

The hierarchy of objectives obtained by AHP could also be used to obtain a scalar set of weights, which could then be applied to a multitude of decision alternatives. While this approach loses some of the accuracy of AHP (Saaty [1987]), the technique is attractive because it provides a means to identify the criteria of importance, and a set of weights for these criteria can be obtained which can then be applied to a large number of alternatives. A limitation of AHP is that the number of pairwise comparisons required can be substantial. Further, the AHP approach would only allow consideration of a maximum of seven alternatives at one time.

Dyer [1990] argued that elicitation questions in AHP aim at determining strength of preference, and thus require subjective estimates on a cardinal scale. Dyer noted that this strength of preference approach has been criticized in the literature. Saaty [1990] responded that AHP has always been understood to be a ratio technique, which can be used to obtain relative measures when absolute measures are not available. AHP, through pairwise comparisons of hierarchical factors, is often used to convert subjective estimates of relative importance into weights. Those articles discuss other points not germane here.

LARGER SCALE SYSTEMS

This paper is concerned with developing weights for multiple criteria in order to support mathematical programming applications, or for supporting expert system applications. Both applications involve large numbers of alternatives available to decision makers, making the pairwise comparison approach less attractive if not impossible. There are many multiple objective mathematical programming techniques available capable of reflecting the features of ordinal utility theory, including Steuer's [1976] method and the method of Zionts and Wallenius [1976]. Steuer's method elicits decision maker preference on the basis of selection from a set of alternatives (usually this set is limited to five alternatives considered at one time). The method of Zionts and Wallenius operates by decision maker pairwise comparison of two alternatives at a time. While both of these approaches are well developed, both have been noted as requiring relatively high degrees of computer support, and neither code is widely available.

Any linear programming package can be used to support multiple objective mathematical programming through weighting the different objectives. These weights must be adjusted for differences in objective scales, but the concept has been around for decades. AHP has been applied in at least three mathematical programming studies. Olson, et al. [1986] considered AHP as a means of obtaining a starting estimate of relative weights for multiple objectives. These weights were adjusted by the technique. Mitchell and Bingham [1986] and Bard [1986] both applied AHP as a means of obtaining relative weights for diverse objectives, and then using these weights to combine objectives into one function which was used in a mathematical programming algorithm to evaluate alternatives. Both of those applications were combinatorial, involving many potential feasible alternatives.

Another application involving many alternatives available to decision makers would arise in applying expert systems to multiple objective choice decisions. Saaty [1988b] presented a means for applying AHP to comparison of large numbers of alternatives. This approach would require absolute measurement of alternative performance rather than ratio measurement. Saaty emphasized that differences in measurement scaling must be considered in this approach in order to avoid diluting important criterion through differences in subcriterion. Because of this need to avoid scaling distortion, Saaty also emphasized the need to rely upon expert assessment of alternative performance, and stated that this approach was not an all purpose approach for ranking large numbers of alternatives.

AHP for Absolute Measurement

Develop Hierarchy	eigenvector of pairwise comparisons
Alternatives any finite	eigenvector of class performance

THE CENTROID METHOD

Solymosi and Dombi [1986] presented a technique which would yield a set of weights for multiple objectives based upon ordinal preference information among pairs of criteria. This approach was intended to be interactive, in that decision makers would make as many preference selections as were necessary to yield acceptable weights. The essence of the technique is that preference information between criteria yields knowledge about the bounds of specific weight values. Individual weights could take on a range of values. Solymosi and Dombi used the centroid of the bounded area as a likely estimate of the true weights implied by preference statements. The basis for this approach is to minimize the maximum error by finding the weights in the center of the region bounded by the decision maker's ordinal ranking of factors. This technique was extended to an AHP framework by Olson and Dorai [1991]. Independently, Rietveld [1989] developed the same technique, with an extension including stochastic dominance.

Once a hierarchy of factors is obtained from phase 1 of AHP, preference information can be obtained as an alternative to the pairwise comparisons. In fact, all decision makers would have to do would be to rank order (with preference or equality) all factors in the hierarchy which did not have subordinate elements. An expected advantage of the centroid approach is that all factors are considered directly. In AHP, a potential problem is that subelements of one branch of the hierarchy are never directly compared to elements in the other branches. As in AHP, the sum of the weights in the centroid approach would add to one (be normalized). Solymosi and Dombi suggest the centroid, obtained by averaging all extreme points, as an estimate of the true set of weights.

For a case involving three factors {A B C} ordinally ranked A > B = C, the limits for weights w_A, w_B and w_C would be [1 0 0] and [1/3 1/3 1/3]. The centroid of the feasible bounds on weights would be [2/3 1/6 1/6]. Note that if all relationships are strict preferences, the set of weights can be determined by formula. For n factors, the weight of each factor will be:

for factor k = 1 to n, $\{\sum_{i=k}^{n} (1/i)\}/n$

A table of values for n factors is appended.

Centroid for Absolute Measurement

Develop Hierarchy		centroid of pairwise comparisons
Alternatives	any finite	0-1 score of class performance

The centroid approach combines the structured means of identifying objective factors provided with AHP, with a straightforward means of obtaining factor weights by utilizing preference information from the decision maker. Solymosi and Dombi proposed an iterative procedure which would elicit preference information until the decision maker was satisfied with the resulting weights. However, by using the AHP approach, more control over collectively exhausting all factors of importance is obtained. Once that is done, simple preference ordering of all factors provides information which can be used to infer weights for each factor in a manner much less involved than the pairwise comparison technique. Of course, the pairwise comparison technique provides the ability to develop more precise weights, but at the cost of potentially many pairwise comparisons.

This approach has been used in a classroom setting, and compared with AHP (Olson and Dorai [1991]). Students applied AHP, as well as provided their ordinal ranking of criteria. Centroid results were compared with AHP results. The post analysis holistic ranking of each subject was used as the basis for comparison, although no absolutely convincing correct answer exists. Viewed in terms of matching the decision (alternative ranked first), the AHP based approach matched the holistic analysis 34 of 46 times (73.9%), while the centroid approach matched the holistic analysis 32 of 46 times (69.6%). This would indicate some obvious value of both the AHP and centroid methods in supporting a decision. While the centroid approach was not as accurate in matching the holistic assessment, there were two cases where the AHP approach did not match the holistic first choice, while the centroid technique did.

COMPARISON OF AHP AND CENTROID FOR MANY ALTERNATIVES

While AHP was intended to provide a ratio scale of relative performance, only seven alternatives can reasonably be compared. Saaty [1988 pp. 264-266] presented a means to apply absolute measurement with AHP to enable experts to evaluate large numbers of alternatives. To demonstrate the centroid approach to AHP, Saaty's example comparing computer stores is used. The hierarchy for this application was:

VALUE

Selection	Atmosphere	Attention	Attitude	Recommended	Service	Price
(.281)	(.029)	(.030)	(.027)	(.049)	(.366)	(.218)

The weights for these seven criteria were developed with the normal pairwise comparison and eigenvalue calculation. Four levels of performance were then considered for each of these seven criterion, again using pairwise comparisons. This set of resultant ratios was then applied based upon expert categorization into these four performance levels of each of the 24 alternatives on each criterion. Saaty emphasized the need for sound expertise in categorizing alternative performance.

Alternative	Selection	Atmosphere	Attention	Attitude	Recommend	Service	Price
1	B	A	A	A	B	A	A
2	A	D	C	C	B	C	A
3	C	C	A	B	B	C	A
4	D	C	B	B	D	C	A
5	C	B	B	B	D	A	B
6	B	B	A	B	C	B	B
7	B	B	A	A	C	C	B
8	C	B	B	B	D	B	B
9	B	A	A	B	C	A	C
10	A	B	A	B	C	C	C
11	C	D	A	B	C	A	C
12	D	A	A	A	C	A	C
13	C	C	B	B	B	B	C
14	C	B	C	B	B	B	C
15	C	D	D	C	B	B	C
16	C	B	C	B	C	C	C
17	B	D	B	A	B	D	C
18	A	C	A	B	C	A	D
19	A	A	D	B	D	A	D
20	B	A	A	A	D	B	D
21	D	D	D	D	C	A	D
22	D	D	D	D	C	A	D
23	B	B	C	C	D	C	D
24	C	B	B	C	D	C	D

A–best B–2nd best C–third best D–worst

To obtain the ratios of the four categories, Saaty used pairwise comparisons, allowing decision-makers to reflect relative importances by criteria.

The centroid approach to this problem would obtain ordinal rankings of the seven criteria. The ordinal ranking implied by the AHP comparisons yields:

Service > Selection > Price > Recommended > Attention > Atmosphere > Attitude

For seven criteria, the bounds on weights would be:

	Service	Selection	Price	Recommended	Attention	Atmosphere	Attitude
	1.0	–	–	–	–	–	–
	.5	.5	–	–	–	–	–
	.33333	.33333	.33333	–	–	–	–
	.25	.25	.25	.25	–	–	–
	.2	.2	.2	.2	.2	–	–
	.16667	.16667	.16667	.16667	.16667	.16667	–
	.14286	.14286	.14286	.14286	.14286	.14286	.14286
Σ	2.59286	1.59286	1.09286	.75952	.50952	.30952	.14286
avg	.37041	.22755	.15612	.10850	.07279	.04422	.02041

Scores could be used to reflect relative performance on each of these seven criteria. Here we use 1.0 for the best category (A), .66667 for the second best (B), .33333 for the third best (C), and 0 for the worst. The logic considered here is that the relative importance of the seven criteria has been estimated by the centroid weights. Used for absolute measurement, these are treated the same as weights, which reflect their relative importance. This relative importance is not further diluted by further breaking down of relative performance.

When this approach is applied to the computer store selection example, the results are:

Alternative	Centroid Score	Centroid Rank	AHP Rank	Δ^2
1	.88798	1	1	0
2	.61054	8	2	36
3	.52891	13	7	36
4	.30442	23	12	121
5	.64195	6	6	0
6	.65477	5	10	25
7	.53810	11	13	4
8	.46644	17	15	4
9	.74093	2	5	9
10	.55510	10	8	4
11	.62086	7	9	4
12	.59604	9	11	4
13	.52404	14	17.5	12.25
14	.51451	15	17.5	6.25
15	.46871	16	20	16
16	.35487	20	22	4
17	.34501	21	23	4
18	.73526	3	3.5	.25
19	.65579	4	3.5	.25
20	.53606	12	14	4
21	.42132	18.5	17.5	1
22	.42132	18.5	17.5	1
23	.33572	22	21	1
24	.28413	24	24	0
				$\Sigma=297$

The focus of this paper is on the use of the centroid as a means of applying the AHP principle to comparison of many alternatives. In order to test the similarity of results, three available AHP applications were taken from the literature. The first application was from Saaty [1988], given above. The other two were Cook, et al. [1989] and Dalal and Thammaneewong [1989], who presented AHP applications with more than seven alternatives. Note that AHP is considered to give decision makers (or experts) greater ability to fine tune relative weights.

AHP rankings were provided by the source authors. These three cases involved from 16 alternatives with seven criteria, 24 alematives with seven criteria, and 117 alternatives over ten criteria. There is no basis for establishing accuracy, because both or neither of the calculation procedures might reflect ultimate "truth". The intent here is to measure the degree of difference in outcome. Problem set rankings were compared using Spearman's ρ. This is a nonparametric test, and does not establish that the two techniques yield equivalent solutions, but rather fails to reject significant difference.

Source	**n**	**Spearman's ρ**	**.999 Limit**	
Cook	16	.9875	.7265	fail to reject difference
Saaty	24	.8709	.6070	fail to reject difference
Dalal	117	.9587	.0928	fail to reject difference

In these results, Spearman's ρ provides a measure for rankings roughly equivalent to correlation. In all three cases, the null hypothesis of mutual independence is rejected at the .999 level (the most difficult tabular value).

CONCLUSIONS

This study sought to compare the pairwise comparison approach and centroid weights in developing a value formula reflecting the relative importance of multiple objectives. The ability to identify such weights is useful in many decision supporting contexts. One example would be in determining the weights for combining multiple objectives in mathematical programming or expert systems.

While the AHP based approach would be expected to be more accurate, the centroid technique has attraction in that it provides nearly as accurate a set of weights, while requiring much less input of a potentially less confusing nature to decision makers. Since most users of decision support packages are not expected to be experts in various techniques, this can be instrumental in the successful delivery of analytic approaches to management. In AHP, decision makers are asked to: 1) identify objectives, as well as subobjectives, and organize them into a hierarchy; and 2) conduct pairwise comparisons at each node of the hierarchy. Step 1) has proven to be a very useful approach, allowing decision makers to concentrate upon what they want to accomplish. The centroid approach in this study utilized step 1) from the AHP analysis, but substituted an ordinal preference ranking of factors for the more involved pairwise comparisons. While pairwise comparisons are not difficult to do, the repetitiveness of the operation may be a burden to some decision makers. Nearly as accurate results (when seeking a set of weights) are available from the centroid approach. Other approaches for obtaining weights are also available, but generally require even more involved analysis than the pairwise comparisons of AHP.

Preference information of the factors reflecting multiple objectives can be identified by identifying the extreme points of the bounds upon individual weights. Ordinal ranking of all factors in one step is required of the decision maker. If no ties are present in this preference ranking, a formula for the individual weights as a function of the number of factors was presented. If ties are present, a simple calculation will yield the centroid of feasible weights.

REFERENCES

Bard, J. F., A multiobjective methodology for selecting subsystem automation options. Management Science 32, 1986, 1628-1641.

Belton, V., "A comparison of the analytic hierarchy process and a simple multi-attribute value function", European Journal of Operational Research 26, 1986, 7-21.

Cook, D. R., S. Staschak, W. T. Green, Jr., and L. G. Vargas, A method to allocate livers for orthotopic transplantation, Proceedings of the International Conference on Multiple Criteria Decision Making: Applications in Industry and Service, Asian Institute of Technology [1989], 779-792.

Dalal, M. and R. Thammaneewong, Business school ranking, Proceedings of the International Conference on Multiple Criteria Decision Making: Applications in Industry and Service, Asian Institute of Technology [1989], 849-864.

Dyer, J. S., "Remarks on the Analytic Hierarchy Process", Management Science 36 3, 1990, 249-258.

Keeney, R. L. and Raiffa, H., Decisions with Multiple Objectives, John Wiley & Sons, New York, 1976.

Krovak, J., Ranking alternative - Comparison of different methods based on binary comparison matrices, European Journal of Operational Research 32 [1987] 86-95.

Mitchell, K. H. and G. Bingham, Maximizing the benefits of Canadian Forces equipment overhaul programs using multi -objective optimization. INFOR 24, 1986, 251-264.

Olson, D. L. and Dorai, V.-K., Implementation of the centroid method of Solymosi and Dombi. European Journal of Operational Research, to be published.

Olson, D. L., M. Venkataramanan and J. Mote, A technique using analytic hierarchy process in multiobjective planning models. Socio-Economic Planning Sciences 20, 1986, 361-368.

Rietveld, P., Using ordinal information in decision making under uncertainty. Systems Analysis Modeling in Simulation 6:9, 1989, 659-672.

Roy, B., "ELECTRE III: Un algorithme de classement fonde sur une representation floue des preferences en presence de criteres multiple", Cahiers du Centre Etudes Recherche Operationelle 20, 1978, 3-24.

Saaty, T. L., A scaling method for priorities in hierarchical structures. Journal of Mathematical Psychology 15, 1977, 234-281.

Saaty, T. L., The Analytic Hierarchy Process, McGraw-Hill, New York, 2nd ed. 1988.

Saaty, T. L., Decision Making for Leaders, Van Nostrand Reinhold, 1982, RWS Publications, 1988.

Saaty, T. L., "An exposition of the AHP in reply to the paper 'Remarks on the analytic hierarchy process' by J. S. Dyer", Management Science 36:3, 1990. 259-268.

Schoemaker, P. J. H. and Waid, C. C., "An experimental comparison of different approaches to determining weights in additive utility models", Management Science 28, 1982, 182-196.

Shim, J.P., Bibliographical research on the analytic hierarchy process (AHP), Socio-Economic Planning Sciences 23, 3, 1989, 161-167.

Solymosi, T. and J. Dombi, A method for determining the weights of criteria: The centralized weights. European Journal of Operational Research 26, 1986, 35-41.

Steuer, R. E., "Multiple objective linear programming with interval criterion weights", Management Science 23:3, 1976, 305-316.

Zahedi, F., The Analytic Hierarchy Process - A survey of the method and its applications. Interfaces 16:4, 1986, 96-108.

Zionts, S. and Wallenius, J., "An interactive programming method for solving the multiple criteria problem", Management Science 22:6, 1976, 652-663.