ISAHP 1996, Vancouver, Canada, July 12-15, 1996

EXPLORATORY ANALYSIS OF MEAN GROUP RANDOM INDICES AND CONSISTENCY IN GROUP DECISION MAKING USING AHP

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Abstract: AHP permits evaluation of a decision maker's consistency by computing a consistency index that is compared with a consistency index computed from random pairwise comparisons. If the resulting (consistency) ratio is less than 0.1, the comparisons are generally said to be consistent. In applying AHP to group decisions combined using the geometric mean, a similar approach is used where the consistency index for the group is compared with the random index for an individual. This comparison may be misleading. In this research, we develop mean random consistency indices for groups of decision makers for possible use in computing group consistency ratios. Specifically, we present the preliminary results of a simulation study, where the mean group random indices (MGRIs) were computed for randomly generated comparison matrices that were combined via the geometric mean. This paper shows that as the group size increases, the values of the MGRIs decrease exponentially.

Introduction

Increasingly, decisions are being made by groups of decision makers. Because of this, the various processes used by groups to arrive at decisions have been studied extensively by researchers including space/time considerations, identification and size of the group of decision makers, and means of incorporating subjective and collective judgments (Huber, 1984; Hatcher, 1992). Many different approaches have been developed to facilitate group decision making. The Delphi method (Linstone and Turoff, 1975) is one of the early approaches that provided a means for incorporating feedback among the group and allowed the decision makers to refine their judgments until an agreement was reached. The Nominal Group Technique (Delbecq and Van de Ven, 1971) is a face to face, but generally noninteracting method for group decision making. Utility based methods (Keeney and Kirkwood, 1975) generate more precise measurements of preferences for decision alternatives which are then aggregated for the entire group. The Analytic Hierarchy Process (AHP) (Saaty, 1980) is another multi-attribute approach for modeling decision making, where the decision process is represented in a hierarchical form. AHP has been commonly used in group decision making, particularly within computerized group decision support systems. In a recent study by Iz and Gardiner (1993), five out of eight of the hierarchical methods used in evaluating discrete alternatives in a group setting employed the use of the AHP.

In most group decision making processes, there is no effective means of measuring the quality of the decision. Many methods drive toward consensus and others use simple voting. In both cases, there is no useable way of measuring the quality of a decision. In the Analytic Hierarchy Process, a measure of consistency has been developed to estimate the degree of (in)consistency that an individual decision maker has with respect to a pairwise evaluation of alternatives or attributes. This inconsistency measure has been applied directly when AHP has been used with a group of decision makers to determine when the group of decision makers is consistent. In working with AHP in group situations, the authors have noted that when this measure is used, larger groups generally display greater consistency. The result appears not to be due to any greater wisdom of the larger group, but rather due to the averaging effect of the geometric mean used to aggregate pairwise comparisons. In these cases, the computed consistency may actually overstate the true consistency. This misleading result is due to the computation of the consistency ratio that compares the computed consistency index for the pairwise comparisons with a random index that represents the consistency index obtained by randomly assigning pairwise comparison values. The random index was generated for individual decision makers and does not account for group (geometric mean) effects.

In this paper, we present the preliminary results of a simulation study, where the mean random indices (MRIs) were computed for randomly generated pairwise comparison matrices that were then combined via the geometric mean for groups of various sizes. Section 2 provides an overview of how the AHP is used in group decision making and describes the use of the geometric mean to combine group responses. Section 3 provides an overview of how consistency is measured in the AHP and describes how the mean random index has been simulated and used in measuring consistency. In section 4, a simulation experiment used to generate mean random indices for random comparison matrices combined via the geometric mean is described. The results of this experiment are presented in section 5, and section 6 contains our concluding remarks.

The Analytic Hierarchy Process and Group Decision Making

In many organizations, decisions are made collectively. In some situations it is difficult for group members to reach consensus or for all of the group members to meet. The Analytic Hierarchy Process has been widely used for group decision making and survey processes. Iz and Gardiner (1993) reported that five out of eight of the hierarchical methods used in evaluating discrete alternatives in a group setting employed the use of the AHP. The AHP has been recommended as the method of choice in integrating criteria and performance measures of different decision makers in military decisions (Hatcher, 1992). Some recent applications employing the AHP for group decisions include selection of a cargo handler (Bard and Sousk, 1990), advertising agency selection (Davies, 1994), selection of a city as a new provincial seat in South Korea (Choi et al., 1994), analysis of consumer bank selection decisions (Javalgi et al., 1989), and parliamentary negotiations (Hamalainen and Leikola, 1995).

There are several ways of obtaining a group response when using the AHP. Dyer and Foreman (1992) discussed four approaches: (1) obtaining *consensus* when members of the group have essentially the same objectives, (2) *voting or compromising* when consensus cannot be reached, (3) using the *geometric mean* of the individual judgments if consensus cannot be reached or the individuals are unable or unwilling to vote, and (4) combining results from individual models or parts of a model if group members have significantly different objectives. The latter approach may involve the use of Delphi and Nominal Group Techniques in conjunction with AHP. The Delphic hierarchy process (Khorramshahgol and Moustakis, 1988) is one such combined approach. It has also been suggested that the minimum and maximum values for each entry in the judgment matrix can be obtained from the group and then the interval AHP can be used to generate the combined group response (Salo and Hamalainen, 1992).

In order to aggregate the individual pairwise comparisons into a single judgment matrix, the reciprocal property which is essential in the AHP, must be preserved. Aczel and Saaty (1983) demonstrated that the geometric mean is the uniquely appropriate rule for maintaining this property. The pairwise judgments of the individual decision makers are combined by computing the geometric mean of the individual pairwise comparisons, and then the resulting comparison matrix is used to derive the ratio scale priorities.

The theory of AHP does not require that decision makers be perfectly consistent when making the pairwise comparisons. Instead it provides a measure of the inconsistency (the consistency ratio), that compares the inconsistency of the judgments made in a given pairwise comparison matrix (the consistency index) to what the inconsistency would have been if the pairwise comparisons were made randomly (the mean random index). A consistency ratio of zero means that the decision maker was perfectly consistent and a value of 1.0 means that the consistency is the same as if the judgments had been made randomly. As a rule of thumb, it is suggested that inconsistency can be considered tolerable only when it is of a lower order of magnitude (10 percent) than the mean random index (Saaty, 1988).

There have been very few studies that provide guidance as to the level of inconsistency that should be considered tolerable in combined group responses. Basak (1988) posited that group judgments must be homogeneous in order for them to be combined and then described an approach to determine when it is appropriate to combine group judgments. She did not, however, try to assess the consistency of the group response. In studies that have used the geometric mean to combine the individual responses, the consistency ratios generated by Expert Choice are often reported. In these studies, the original mean random indices (MRI) have been used in the denominator of the Consistency Ratio to evaluate the consistency of the group response. However, as we will show, the use of these MRIs may greatly overestimate the consistency for the group response.

Measurement of Consistency and the Mean Random Index (MRI)

The AHP allows for inconsistencies in the pairwise evaluations of individual decision makers. The sources of inconsistency can be divided into two groups: (1) the inconsistency may be inherent in the decision maker's preferences, or (2) the inconsistency may be due to the way in which the decision maker's preferences are elicited (Dadkhah and Zahedi, 1993). In the latter case, inconsistency is viewed as a measurement error and thus, if identified, can be reduced. Both Harker (1987) and Dadkhah and Zahedi (1993) provided approaches for identifying and reducing the inconsistencies. There are a variety of causes for inconsistencies including clerical errors, lack of information, a lack of focus or concentration during the judgment process, randomness, and inadequate problem structure (Dyer and Foreman, 1992).

In the AHP, the decision maker assigns relative preferences or importances as to the degree to which one attribute (or alternative) A_i dominates another attribute (or alternative) A_j. These importances (given on a specified scale--often the 1 to 9 scale due to Saaty) are captured in a matrix of pairwise comparisons **A**, where the ij^{th} entry of the matrix, a_{ij}, is an estimate of the ratio of the importance weights of the two attributes, w_i/w_j. The resulting matrix is a positive reciprocal matrix having the following properties: (1) $a_{ij} = 1/a_{ji}$, (2) $a_{ii} = 1$, and (3) $a_{ij} > 0$ for all i and j.

If the decision maker is perfectly consistent, then $a_{ik} = a_{ij}*a_{jk}$. For example, if the decision maker reports that $a_{ij} = 3$ and $a_{jk} = 2$, then the decision maker must say that $a_{ik} = 6$ if the decision maker is to remain perfectly consistent. Jensen and Hicks (1993) described the need for ordinal or cardinal transitivity in paired comparisons in order to develop an unambiguous ranking or ratio scaling of the preferences.

If $\underline{w}$ is the vector of weights and the decision maker is perfectly consistent, then any one row of the pairwise comparison matrix would be sufficient to determine $\underline{w}$. In particular, the weights are determined by solving the following equation for $\underline{w}$:

$$\mathbf{A}\underline{w} = \lambda\underline{w} \tag{1}$$

where λ is the right eigenvalue and $\underline{w}$ is the associated right eigenvector. In the perfectly consistent case, the matrix **A** has rank one and there is only one positive nonzero eigenvalue.

It has also been shown that under perfect consistency the value of the right eigenvalue is equal to n, the dimension of the matrix **A** (Saaty, 1977; Dadkhah and Zahedi, 1993). When the decision maker is not perfectly consistent, then it has been shown that the comparison matrix will have a maximum right eigenvalue λ_{max} that is greater than n (Saaty, 1977) and that estimates of the weights are given by the associated right eigenvector.

Saaty suggested a metric for measuring inconsistency based on the value of λ_{max}. Specifically, the consistency index, CI(n), is a normalized measure of the deviation of the maximum right eigenvalue for a given comparison matrix from n (the value obtained for perfect consistency).

$$\mathrm{CI}(n) = \frac{\lambda_{max} - n}{n - 1} \tag{2}$$

The consistency index has a value of zero in the case of perfect consistency. In order to establish a threshold for acceptable levels of inconsistency, the CI(n) was compared to the average CI(n)s (called mean random indices--MRI(n)) for a large number of randomly generated matrices of dimension n. The MRI(n)s can be computed from the maximum eigenvalues as follows:

$$\mathrm{MRI}(n) = \frac{\overline{\lambda}_{\max} - n}{n-1} \tag{3}$$

where $\overline{\lambda}_{\max}$ is the average of the principal right eigenvalues for the randomly generated matrices.

Based on the simulated MRI(n) values, Saaty has defined the consistency ratio to be

$$\mathrm{CR}(n) = \mathrm{CI}(n)/\mathrm{MRI}(n) \tag{4}$$

where CR(n) = 0 would imply perfect consistency and a value of CR(n) = 1 would imply that the consistency is the same as if the judgments had been made randomly.

Other measures of consistency have been suggested by researchers, particularly for some of the variations on the AHP. For example, Jensen and Hicks (1993) described a coefficient of consistency to use with ordinal judgments and state that this approach can be applied to ratio scaled paired comparison matrices to test for ordinal response inconsistencies. Wedley (1993) described the use of regression equations to predict the consistency index for incomplete AHP comparison matrices. Salo (1993) described a method for modeling the inconsistency of a decision maker's judgments based on the local priorities. The approach allows consistency to be evaluated for both incomplete and/or interval entries in the comparison matrices. Another form of inconsistency, called inverse inconsistency, is described by Dodd et al. (1995). They developed a measure of inverse inconsistency based on the difference between the dominant right eigenvector and the inverse of the dominant left eigenvector and suggest that this measure be used in conjunction with the consistency ratio. The consistency ratio given in equation (4) remains the most commonly used measure of consistency in the AHP.

In order to apply the CR(n) in practice, randomness is used to reject the hypothesis of consistency (Dadkhah and Zahedi, 1993). Saaty originally stated that an acceptable consistency ratio should be less than 0.1 (less than 10% of the MRI(n)). If the CR(n) value exceeds 0.1, then the pairwise comparison matrix should be examined for inconsistencies and re-evaluated. Vargas (1982) showed that 10% was an acceptable upper bound. Some researchers have shown that there is a relationship between the acceptable percentage and the size of the matrix. For example, Lane and Verdini (1989) investigated the distribution of random inconsistency and the implications on Saaty's decision rule. They suggested stricter consistency requirements for $n = 3$ and 4. Others have suggested that the 10% rule may be too strict (see, for example, Apostolou and Hassell, 1993). The current recommendations are that the consistency ratio should be about 5% or less for a 3×3 matrix, 8% or less for a 4×4 matrix, and the 10% rule applies for higher order matrices (Saaty, 1994).

There have been a number of values for MRI(n)s reported in the literature. Table 1 provides a summary of the reported values. Saaty (at Wharton) and Uppuluri (at Oak Ridge) were the first to perform simulation experiments with 500 and 100 runs, respectively (Saaty, 1988). Lane and Verdini (1989), Golden and Wang (1989), Noble (1990), Donegan and Dodd (1991), and Tummala and Wan (1994, 1992) followed these original experiments. The most recent study by Tummala and Wan (1992, 1994) used a designed experiment to control for both the number of runs and the random number seed. They used the Power Method to determine the largest eigenvalue which is an approach similar to Saaty's method. Saaty (1988) found the principal right eigenvalue and the associated eigenvector by raising the pairwise comparison matrices to increasing powers and estimating the weights as:

$$\underline{\mathrm{w}} = \lim_{n\to\infty} \frac{A^n \underline{e}}{\underline{e}^T A \underline{e}} \tag{5}$$

where A is the pairwise comparison matrix and $\underline{e}$ is a vector consisting of ones in each entry. The Power method, which is numerically more stable, normalizes the trial vector at every iteration, while Saaty's method normalizes only at the last iteration. Although a number of researchers have repeated the original experiments to generate the MRI(n)s, the boldfaced values in the last two columns are still the most commonly used MRI(n)s reported in the literature. All of the results in Table 1 were generated using Saaty's 1 to 9 scale.

Table 1: Mean Random Indices Generated in Previous Studies

Experiment	Tummala and Wan (1994)	Noble (1990)	Lane and Verdini (1989)	Donegan and Dodd (1991)	Golden and Wang (1989)	Wharton Saaty (1988)	Oak Ridge Saaty (1988)
# of runs	4,600 - 470,000	5,000	2,500	1,000	1,000	500	100
3	0.500	0.49	0.52	0.4887	0.5799	*0.58*	0.382
4	0.834	0.82	0.87	0.8045	0.8921	*0.90*	0.946
5	1.046	1.03	1.10	1.0591	1.1159	*1.12*	1.220
6	1.178	1.16	1.25	1.1797	1.2358	*1.24*	1.032
7	1.267	1.25	1.34	1.2519	1.3322	*1.32*	1.468
8	1.326	1.31	1.40	1.3171	1.3952	*1.41*	1.402
9	1.369	1.36	1.45	1.3733	1.4537	*1.45*	1.350
10	1.406	1.39	1.49	1.4055	1.4882	*1.49*	1.464
11	1.433	1.42	n/a	1.4213	1.5117	*1.51*	1.576
12	1.456	1.44	1.54	1.4497	1.5356	n/a	*1.476*
13	1.474	1.46	n/a	1.4643	1.5571	n/a	*1.564*
14	1.491	1.48	1.57	1.4822	1.5714	n/a	*1.568*
15	1.501	1.49	n/a	1.4969	1.5831	n/a	*1.586*

There have been very few studies that provide guidance as to the level of inconsistency that should be considered tolerable in combined group responses. In studies that have used the geometric mean to combine the individual responses, the consistency ratios generated by Expert Choice are often reported. In these studies, the original mean random indices (i.e., MRI(n)s in the last two columns of Table 1) have been used to evaluate the consistency of the group response. The following experiment estimates the values of the mean random indices for the group response combined via the geometric mean. It shows that the random indices decrease with group size, indicating that the larger the group size, the more consistent the random group response.

A Simulation Experiment for Generating Mean Group Random Indices–MGRI(n,m)

The original random indices were based on the assumption that a single decision maker was making random responses when performing the pairwise comparisons. Our investigation examines what happens when several individuals make their responses randomly, and then these responses are combined via the geometric mean. We were interested to see what happens to the random index as the size of the group increases. A preliminary investigation of the impact of group size m on the value of the random indices for matrices of size n was conducted. In order to compute the mean group random index, MGRI(n,m), for groups of size m and comparison matrices of order n, the following general approach was used. Saaty's 1 to 9 scale is used that results in 17 possible realizations of pairwise comparisons (1/9,1/7,...,1,2,...9). First, m $n \times n$ comparison matrices using the 17 equally likely values from Saaty's scale are generated at random and then the geometric mean of the corresponding entries are computed to obtain the combined random group response. Then the maximum eigenvalue is computed for this matrix. This experiment is repeated N times, in order to estimate the mean value of the maximum eigenvalue. This value is then used in equation (3) in order to compute MGRI(n,m).

The simulation study examined matrices of order n = 3 to 9 and group sizes of m = 1 to 5, 7, 10, 15, 20, 25, 30, 50, 75, and 100. The program was written in GWBasic and the random generator included with GWBasic was used to generate uniform random variates. The following steps were used in the experiment:

STEP 1

A matrix of the desired order was generated by generating a uniform random number from 1 to 17 for each element of the matrix and assigning the respective number from the set

$\{1,2,3,4,5,6,7,8,9,1/2,1/3,1/4,1/5,1/6,1/7,1/8,1/9\}$. The seed for the random number was set equal to the clock time.

STEP 2

For group size m, m random matrices were generated in one replication and the geometric mean for each element of the m matrices was computed. The weights for this matrix were calculated as

$$w_i = \frac{\sum_{j=1}^{n} a_{ij}}{\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}} \quad \text{for } i = 1,2,\ldots,n \tag{6}$$

STEP 3

The matrix of geometric means was squared and the weights of this matrix were calculated as in equation (6).

STEP 4

Each of the elements of the two weight vectors were compared and if the differences were equal to or less than 10^{-2} for all elements, then STEP 5 was completed. If the difference between any elements of the two weight vectors was greater than 10^{-2}, then the iterative procedure was continued (i.e., the squared matrix was squared again and the new weights were computed and then compared with the weights from the previous iteration.)

STEP 5

The value of λ_{max} was estimated by finding the row sums of the geometric mean matrix and multiplying it by the final weights.

STEP 6

The group random consistency index, GRI(n,m), was computed using the λ_{max} from STEP 5 in equation (3).

STEP 7

Steps 1 to 6 were replicated 50 times and the sample mean group random index was computed. The whole experiment was repeated 20 times in order to find the MGRI for 1000 replications. Thus 20 samples of 50 randomly generated geometric mean matrices were used to generate the results presented in Section 5.

Several preliminary runs were used to determine a sample size at which the mean group random index appeared to reach steady state. Figure 1 shows the results for one of the preliminary runs where $n = 4$ and $m = 7$. From these initial runs, it was felt that 1000 replications would be sufficient to evaluate whether or not the size of the group had an impact on the value for the random index for the combined group response.

Simulation Results for MGRI(n,m)

Table 2 and Figure 2 show the results of our preliminary investigation using N = 1000 replications of the simulation. The first row of Table 2 contains our simulated Mean Random Indices for individual responses and should replicate the other experimental values given in Table 1. Note that our values tend to be slightly on the high side compared with Tummala and Wan's (1994) values, and with the exception of $n = 6$, lie within the upper and lower values reported in prior studies. Each of the remaining rows of the table indicates what happens to the mean random index as the group size increases.

Figure 1: Determination of Number of Replications

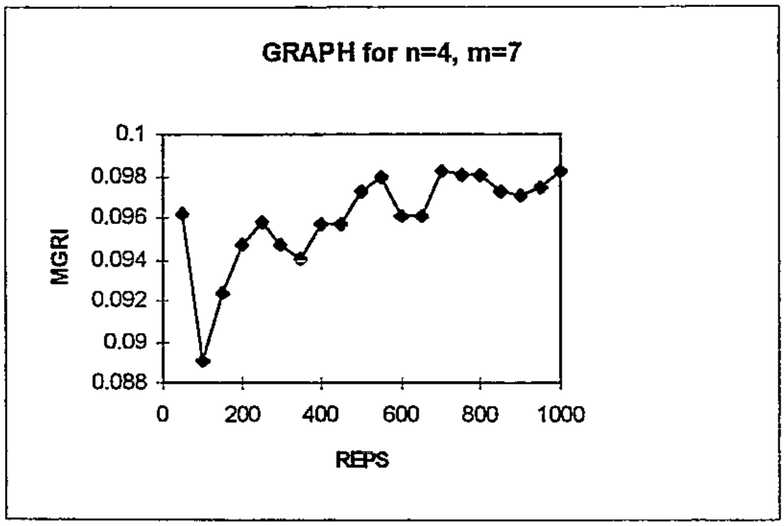

Table 2: Simulation Results for the Mean Group Random Indices MGRI(*n*,*m*)

Group Size	MGRI for n=3	MGRI for n=4	MGRI for n=5	MGRI for n=6	MGRI for n=7	MGRI for n=8	MGRI for n=9
1	0.519263	0.859056	1.114559	1.267523	1.35928	1.404906	1.436302
2	0.240743	0.38611	0.498194	0.53942	0.610161	0.633248	0.664307
3	0.159285	0.253283	0.312791	0.348876	0.372465	0.401335	0.414484
4	0.115453	0.179372	0.228411	0.251851	0.27465	0.286686	0.297256
5	0.082876	0.136669	0.179919	0.195406	0.212172	0.225971	0.232182
7	0.067983	0.098209	0.122427	0.137659	0.146033	0.152618	0.16077
10	0.044219	0.0693	0.082279	0.090118	0.098307	0.10384	0.109594
15	0.035376	0.045098	0.052605	0.061102	0.064026	0.067443	0.070589
20	0.024795	0.032668	0.039204	0.045509	0.048771	0.051415	0.053513
25	0.018709	0.026126	0.031726	0.035751	0.039433	0.041089	0.042359
30	0.014922	0.022272	0.025811	0.029689	0.031843	0.033155	0.034807
50	0.014451	0.012779	0.015861	0.01786	0.018717	0.019699	0.020414
75	0.008187	0.008631	0.010557	0.012296	0.018662	0.013225	0.013666
100	0.004474	0.006616	0.007794	0.008909	0.009363	0.009969	0.010039

Although our results are only preliminary and we plan to repeat our experiment using the Power Method and more control over the number of replications, we do believe that they indicate some interesting trends with respect to group size. Namely, as suggested in Figure 2, the MGRIs appear to decrease exponentially with group size.

We fit the following model to the data in Table 2.

$$MGRI(n,m) = a_n e^{-b_n m} \qquad (7)$$

We transformed each entry in Table 2 using the natural logarithm and computed the regression equation. The results in Table 3 were obtained (each with an R-squared value of approximately 0.72 and significance level of .00013).

The results indicate that as we combine random responses via the geometric mean, the individual inconsistencies, as measured by the consistency index, tend to diminish. If we would like to use randomness in order to reject the hypothesis of consistency, much smaller values for the consistency index for the combined group response would be required. For example, if the group size was five and the order of the matrix was 6, then a consistency index of less than approximately 0.02 would be required if Saaty's 10% rule were applied directly to the figures in Table 2. In comparison, without taking into account the group size, a consistency index less than 0.12 would be considered to be acceptable.

Figure 2: Simulation Results for Mean Group Random Indices

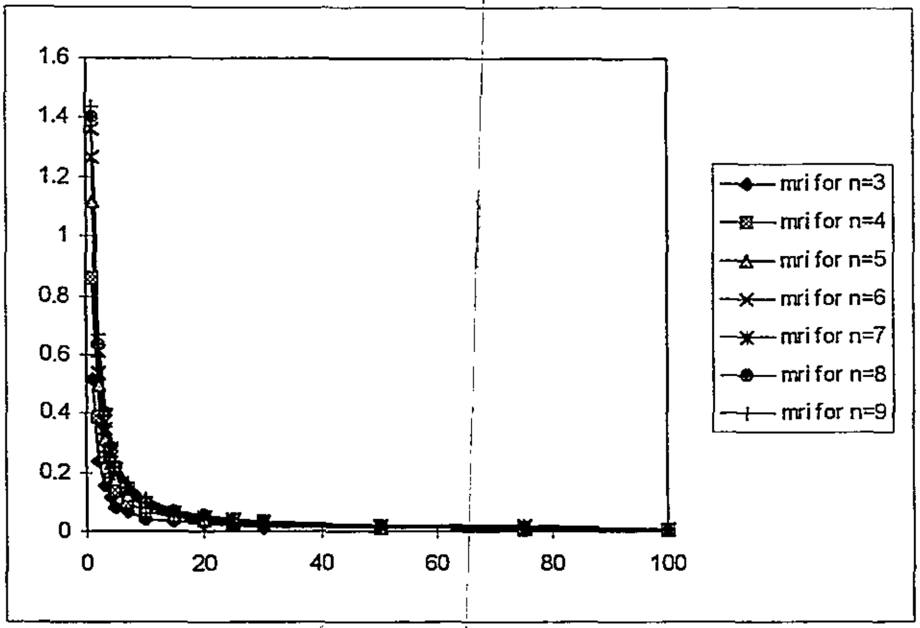

Table 3: Regression Parameters

Order of the Matrix, n	b_n	$\ln(a_n)$
3	-0.038	-2.20
4	-0.041	-1.17
5	-0.042	-1.56
6	-0.041	-1.45
7	-0.040	-1.39
8	-0.042	-1.32
9	-0.042	-1.28

Conclusions

In this paper, we have presented the preliminary results of a simulation study, where the mean group random indices (MGRIs) were computed for randomly generated pairwise comparison matrices that were then combined via the geometric mean. This paper has demonstrated that the MGRI(n,m) values for the group response decrease exponentially with group size (m). The implication is that the random indices commonly used in computing consistency ratios may be misleading when evaluating the group consistency from individual response matrices that have been combined via the geometric mean. Stricter rules for consistency may be required for groups than for individuals if one is trying to avoid the levels of inconsistency that a group of individuals giving random responses would have.

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