

**STUDY ON THE REASONABLE DIVISION OF LABOUR  
AMONG RAIL MARSHALLING YARDS BASED ON AHP**

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**ABSTRACT**

The choosing plan of reasonable division of labour among rail marshalling yards is a decision and ranking problem which needs a kind of common measure. AHP is a method of giving that measure and a decision. It is used to assist a decision maker in choosing plans. According to the AHP theory and set theory, AHP model of practicality, synthesis about the reasonable division of labour among rail marshalling yards is constructed. A method of building GAHP judging matrix is suggested in accordance with the principle of acquiring as much information from the decision makers as possible. Linear function with unified parameters using the ranking of plans are defined. Finally, an example of the ranking and choosing plans of reasonable division of labour among rail marshalling yards for a certain district rail network in China is given.

**I Introduction**

The reasonable division of labour among rail marshalling yards is an important subject which solves the problem of marshalling yards' distribution in the railroad network. It is the promise that the flows of wagons are exactly organized into groups and the daily transport production can be assured. It can also give full play to productive capacity of the rail transportation. At present, several methods that meet the problem have been presented from three different aspects of saving wagon hours, reasonable load of capacity and transport economic profit. There are their own reasons and particular emphases in each methods. To actual rail road, different district and rail network, we could get different plan without a common synthetical standard to measure them.

There always exist a great many factors such as various indices and importance of a plan. The factors are interrelated and interacted each other all the way of choosing the plan of reasonable division of labour. This method seems to be a synthetical evaluation as same as rank under the same standard.

AHP, a kind of practical, available method, makes it possible to solve the problem of the reasonable division of labour among rail marshalling yards.

According to the principle of AHP, combining actual rail network condition of a certain in China, the subject of reasonable division of labour among rail marshalling yards is studied preliminary. Under the basis of analysing every factor and every index dealing with that district rail network, an AHP model of practicality, synthesis about the reasonable division of labour among rail marshalling yards is constructed. Using the idea of Group AHP, a method of building the group judging matrix is suggested in accordance with the principle of acquiring as much information from the quantitative decision makers as possible. Linear function with unified parameters is defined based on Set Theory. Finally, the evaluation and rank for a few of plans is done.

**II. The AHP Model of the Reasonable Division of Labour Among Rail  
Marshalling Yards**

The model constructed previously (see Fig. 1) is composed of four levels.

The object level: the result of choosing plan of the reasonable division of labour among rail marshalling yards.

The criterion level: there are three different aspects of criteria in the level. They respectively meet the demands of the project. Under this level, a sublevel including nine indices appears.

At the bottom is the alternative level including all possible plans.

The criterion of saving wagon hours was usually used in almost all existing methods to choose the plan of reasonable division of labour among rail marshalling yards. So the first aspect of criterion is wagon hours. It includes rearranging wagon hours (B1), collecting wagon hours (B2), capacity of organizing into groups (B8), wagon travelling kilometers (B8), usefulness workload in marshalling yards (B9), the rate of no rearranging wagons (B4), the through rate of wagon flow (B3). Besides, for a certain rail road the reasonable load of its existing equipments should be taken into account before they are improved when choosing plan. The second aspect of criteria is the reasonable load of capacity. It includes six indices, i.e. the through rate of wagon flow (B3), the rate of no rearranging wagons (B4), capacity through station (B5), capacity of organizing into groups (B6), capacity through rail block (B7), usefulness workload in marshalling yards (B9).

The last aspect of criteria is transport economic profit. This criterion reflects the influence of rail transportation upon the state economy. The transport economic profit is one of not only whole rail network but also the certain rail station as well. It includes six indices, i.e. rearranging wagon hours (B1), collecting wagon hours (B2), the rate of no rearranging wagons (B4), the through rate of wagon flow (B3), wagon travelling kilometers (B8), and usefulness workload in marshalling yards (B9).

Obviously, to a certain railroad or rail network, it is impossible that there are common weights of those criteria and their indices. Their weights of relative importance (weight vector) can be decided by combining that rail network and the decision information given by a group of makers. Group AHP judging matrix is built through comparing one thing with another based on AHP, and then their weights can be got by using the method of solving maximum matrix eigenvalue and corresponding eigenvector.

Following (see Fig. 1) is an AHP model of reasonable division of labour among rail marshalling yards.

### III. Group AHP Judging Matrix

According to the AHP theory, every decision maker can give a corresponding judging matrix in accordance with the demand of problem. Each element of matrix can be decided by comparing one thing with another. But, in fact, the reasonable weight of index is not able to be decided only by one maker. In general, a number of judging matrices can be given by a group of makers. So, all of the judging information must be obtained synthetically. A group AHP judging matrix is built.

Suppose that under the standard of the same criterion, there are "N" decision makers in the case. N judging matrices can be given. The Sth maker gives matrix "A<sub>S</sub>".

$$A_g = \begin{bmatrix} a_{11s} & a_{12s} & \dots & a_{1ns} \\ a_{21s} & a_{22s} & \dots & a_{2ns} \\ \dots & \dots & \dots & \dots \\ a_{n1s} & a_{n2s} & \dots & a_{nns} \end{bmatrix}$$

Group AHP judging matrix "A" is:

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}.$$

To the element  $a_{ij}$ , there is:  $a_{ij} \in R$

$$R = \{1/9, 1/8, 1/7, \dots, 1/2, 1, 2, \dots, 9\}.$$

Suppose that  $k_1 < N$  makers give the element " $a_{ij1}$ ";

$k_2 < N$  makers give the element " $a_{ij2}$ ";

⋮

$k_m < N$  makers give the element " $a_{ijm}$ ".

Considering the ratio of " $k_k$ " makers in " $N$ ", it is:

$$\beta_k = \frac{k_k}{N} \quad \sum_{k=1}^m \beta_k = 1$$

Definition:  $f(x)$  is a strictly monotone function and  $a_{ij} = (w_i/w_j) \varepsilon_{ij}$

$$\lim_{\varepsilon_{ij} \rightarrow 1} |f(a_{ij}) - f(w_i/w_j)| = 0$$

According to weighted Hamming--Minkowski distance formula, there is:

$$D = \left( \sum_{k=1}^m [\beta_k (f(a_{ijk}) - f(a_{ij}))]^q \right)^{1/q}$$

minimize  $D$ , i.e.

$$\min(D) = \min \left( \sum_{k=1}^m [\beta_k (f(a_{ijk}) - f(a_{ij}))]^q \right)^{1/q}$$

and then, the solution of function " $f(a_{ij})$ " can be got. The element " $a_{ij}$ " of group AHP judging matrix can be expressed as:

$$a_{ij} = f^{-1}(a_{ij})$$

Suppose the  $x_1$  and  $x_2 \in R$  are two adjacent elements in set " $R$ ", and  $x_1 < x_2$ .

If  $|x_1 - a_{ij}| < |a_{ij} - x_2|$ , then  $a_{ij} = x_1$ , otherwise,  $a_{ij} = x_2$ .

The group AHP judging matrix given by above process can obtain the judging information in accordance with the principle of acquiring as much information from those makers as possible. About the strictly monotone function, it is selected in a certain condition.

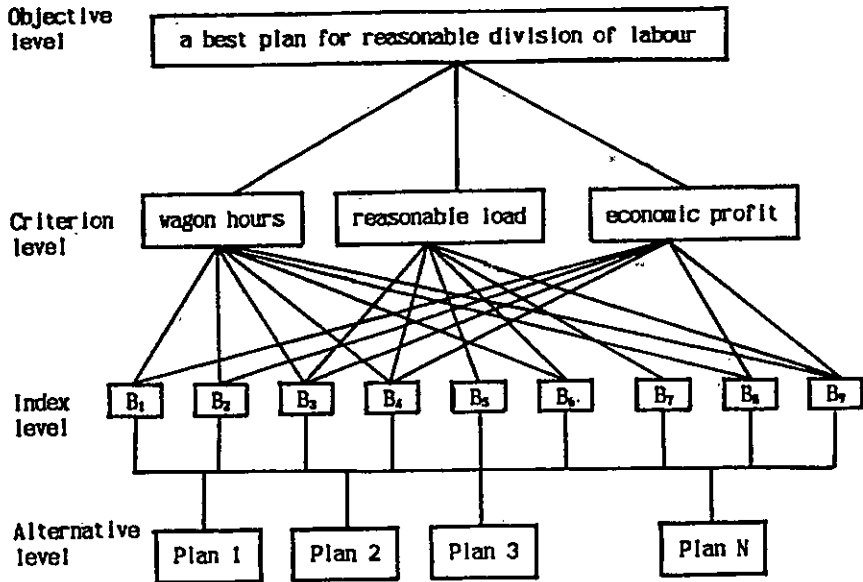


Figure 1: AHP model for the reasonable division of labour among marshalling yards.

#### IV. The Evaluating Weight of Index

According to relative importance of each index judged by makers, through synthetically analysis, group AHP judging matrix can be constructed. The rank weight of each criterion and its indices can be decided by solving the maximum eigenvalue and corresponding eigenvector of GAHP matrix.

By using the AHP model (see Fig. 1) and GAHP matrix, the following is a calculating example for rail network of the certain district in China.

(1). GAHP judging matrix A-C:

A	C1	C2	C3	eigenvector
C1	1	3	5	0.637
C2	1/3	1	3	0.258
C3	1/5	1/3	1	0.105

consistency ratio : C.R = 0.033

( 2 ). GAHP judging matrix  $C_1$ —B1

C1	B1	B2	B3	B4	B6	B8	B9	eigenvector
B1	1	1	3	3	5	7	7	0.288
B2	1	1	3	5	5	7	7	0.318
B3	1/3	1/3	1	1	3	7	5	0.148
B4	1/3	1/5	1	1	5	3	4	0.118
B6	1/5	1/5	1/3	1/5	1	1/5	1/3	0.032
B8	1/7	1/7	1/7	1/3	5	1	1	0.052
B9	1/7	1/7	1/5	1/4	3	1	1	0.044

consistency ratio : C.R = 0.097

( 3 ). GAHP judging matrix  $C_2$ —B1

C2	B3	B4	B5	B6	B7	B9	eigenvector
B3	1	1	1/5	1/7	1/5	1/7	0.034
B4	1	1	1/5	1/7	1/5	1/7	0.034
B5	5	5	1	1	1	1/5	0.138
B6	7	7	1	1	1	1/5	0.160
B7	5	5	1	1	1	1/5	0.138
B9	7	7	5	5	5	1	0.496

consistency ratio : C.R = 0.057

( 4 ). GAHP judging matrix  $C_3$ —B1

C3	B1	B2	B3	B4	B8	B9	eigenvector
B1	1	1	1/5	1/5	3	1	0.087
B2	1	1	1/5	1/5	3	1	0.087
B3	5	5	1	1	7	3	0.349
B4	5	5	1	1	5	3	0.335
B8	1/3	1/3	1/7	1/5	1	1/3	0.041
B9	1	1	1/3	1/3	3	1	0.102

consistency ratio : C.R = 0.023

5) Rank by merging the weight of each index ( $\alpha_i$ )

level C \ level B	C1	C2	C3	composite weight of index
	0.637	0.258	0.105	
B1	0.288	0	0.087	0.193
B2	0.318	0	0.087	0.212
B3	0.148	0.034	0.349	0.131
B4	0.118	0.034	0.335	0.140
B5	0	0.138	0	0.037
B6	0.032	0.160	0	0.062
B7	0	0.138	0	0.036
B8	0.052	0	0.041	0.037
B9	0.044	0.496	0.102	0.167

V. The Plan Rank of the Reasonable Division of Labour Among Rail Marshalling Yards:

(1) Definition--Linear function with unified "n" parameters.

Suppose that variable  $X_i \in [0,1]$ ,  $i \in I$ , parameter  $\alpha_i \in [0,1]$  and  $\sum_{i=1}^n \alpha_i = 1$  and then, the following function

$$Y = \sum_{i=1}^n \alpha_i X_i$$

is named linear function with unified "n" parameters. It is expressed as  $f_p^n$ . It is easy to be demonstrated that  $f_p^n$  is full mapping in  $[0,1]$ .

(2) Conversion of index value

In actual case, absolute index value is often given, but it does not meet the demand of  $f_p^n$ . To make index value meet the demand of  $f_p^n$ , i.e. taking their relative index values, its formula is:

$$b_i = \frac{B_i - B_i'}{B_i^* - B_i'}$$

where the meaning of the symbols in formula is

- $b_i$ : relative value of the ith index;
- $B_i$ : absolute value of the ith index;
- $B_i'$ : minimum of absolute value of the ith index;
- $B_i^*$ : maximum of absolute value of the ith index.

(3) Plan rank

Every plan of the reasonable division of labour among rail marshalling yards can be expressed as an index vector which includes nine indices. Index vector of relative value is

$$\vec{b} = (b_1, b_2, \dots, b_9)$$

The corresponding weight of each index is given by rank by merging. Weight vector is expressed as:

$$\vec{\alpha} = (\alpha_1, \alpha_2, \dots, \alpha_9)^T, \quad \sum_{i=1}^9 \alpha_i = 1$$

According to  $f_p^n$  definition, the synthetically evaluating value of plan is

$$y = \vec{b} * \vec{\alpha} = \sum_{i=1}^9 \alpha_i b_i$$

It is easy to know, the more the y value, the better the plan. The plan that is hoped is :

$$\max(y) = \max \left( \sum_{i=1}^9 \alpha_i * b_i \right)$$

If there are "n" plans in that rail network, i.e.  $b_1, b_2, \dots, b_n$ , their evaluating values obtained by calculating  $(f_p^n)$  are:

$$(y_1, y_2, \dots, y_n)$$

suppose that

$$y_1^* = \max[y_1, y_2, \dots, y_n]$$

$$y_n^* = \min[y_1, y_2, \dots, y_n]$$

The total rank of plans is expressed as:

$$(y_1^*, y_2^*, \dots, y_n^*)$$

and

$$y_1^* > y_2^* > \dots > y_n^*$$

(4) An example of the ranking of a few of plan

For the rail network of the certain district in China, a few of plans about the division of labour among rail marshalling yards are formed because of the changing of route of wagon flow and way of organizing wagon flow. The synthetical evaluation for the following "5" plans is done based on AHP. They are ranked in accordance with the priority of plans.

index weight	0.193	0.212	0.131	0.140	0.037	0.062	0.036	0.037	0.167	evaluating
relative value	b1	b2	b3	b4	b5	b6	b7	b8	b9	value
plan1	0.65	0.43	0.50	0.60	0.70	0.80	0.80	0.67	0.80	0.6306
plan2	0.70	0.60	0.63	0.50	0.80	0.85	0.80	0.65	0.90	0.7003
plan3	0.50	0.70	0.70	0.44	0.75	0.73	0.90	0.70	0.73	0.6514
plan4	0.55	0.67	0.75	0.45	0.85	0.75	0.91	0.62	0.78	0.6734
plan5	0.40	0.80	0.80	0.43	0.90	0.70	0.93	0.60	0.88	0.6912

The best plan is plan2 from the result by calculating. The ranking of plans is: [ plan2, plan5, plan4, plan3, plan1].

#### Conclusion

In the rail network, we first use the AHP theory as a method of choosing plan for the division of labour among rail marshalling yards, and find that AHP is a practical, available method. Its measure of the ranking of plans based on AHP is reliable. But, it is only a first step. With the improvement of AHP theory, especially, GAHP theory. A number of fields in the transport system will widely make use of it in theory. A number of fields in the transport system will widely make use of it in order to get the better decision.

#### References

- [1] Satty, T.L. Concepts, Theory and Techniques Rank Generation, Preservation, and Reveal in the Analytic Hierarchy Decision Process. Desion Science. Vol. 18.
- [2] Satty, T.L. & Vargas, L. Inconsistency and rank preservation. Journal of Mathematical Psychology, 1984. 28.
- [3] Liu Bao & Xu Shubo, "The application of AHP in China and its development", Mathematical Modelling, Vol. 7, No. 9, 1987
- [4] Wang Yanzhang, Aquantitative analysis method of informal systems, Journal of system engeneering, No.1, 1987