DISCUSSION AND PRACTICE ON DETERMINATION OF THE WEIGHT BY AHP FOR DEVELOPMENT OF WATER RESOURCES

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ABSTRACT

Determination of the weight is a very important and difficult problem in the multi-objective decision making and comprehensive evaluation. In this paper, the authors review the various current methods for determination of the weight and analyze the merit and demerit for each method and emphatically discuss the application for determination of the weight by AHP. At the same time, the following problems are also discussed: the method of group AHP; the problem of FUZZY AHP and the check of the consistency.

This paper enumerates several examples:
1.the comprehensive evaluation of water conservancy project and hydro-power engineering by Fuzzy mathematics and AHP. The influential factors of evaluation are divided into three aspects and 12 indexes: the economic and financial effects (4 indexes); the social and political effects (4 indexes) and the environmental and ecological effects (4 indexes).

2. The comprehensive evaluation of management and adiministration for a multipurpose reservoir. The factors are divided into four aspects and 20 indexes; the production management (6 indexes); the realization degree of the project goal (4 indexes); the comprehensive utilization of water and soil resources (3 indexes) and the management effects (7 indexes).

3. The optimum allocation of water resources for economic region. The influential factors are divided into five aspects and 18 indexes: the political effects (6 indexes); the environmental effects (4 indexes) and social effects (5 indexes)etc.They are interlocked to each other.

I. INTRODUCTION

Determination of the weight is a very important and difficult problem in the multiobjective decision making and comprehensive evaluation.

1. The weight problem in the multiobjective optimization

Water resources system is a large and complex system which combines natural system and artificial system in many aspects. The development of water resources influence the national economy , and many objectives must be taken into account, such as the development of both the national economy and the local (regional) economy, the improvement of environmental conditions and the increase of social welfare etc.

If there are p objectives for development of water resources, the multiobjective decision making problem can be generally expressed as

$$\max f(x)=\max_{x\in R}[f_1(x), f_2(x),\dots,f_p(x)] \quad \dots \dots \quad (1)$$

Where $R \triangleq [x \mid g_k(x) < 0, \ \forall k; \ x \geq 0, \ x \in E^n]$

$f(x)$ is p-dimentional vector of objective function

$g_k(x)$ is constraint condition ($k = 1, 2, \ldots, K$)

x is n-dimensional decision vector

R is decision space (fessible region for x)

The method of weighting is one of the commonly used methods for solving multiobjective decision making problem. In this method, we can transform the problem from vector optimization (1) into scalar quantity optimization (2) as follows:

$$p(w): \quad \max \sum_{i=1}^{p} w_i f_i(x) = w^T * f(x)$$

Where w_i is weight coefficient, which reflects the relative importance of each objective, $W_i > 0$ and $\sum W_i = 1.0$. The key is how to determine weight coefficient $w = (w_1, w_2, \ldots, w_p)$ in the method.

2. The weight problem of fuzzy comprehensive evaluation for water conservancy project

The planning, designing, operation and management of a water conservancy project are also a large-scale complex system. The factors to be considered for decision making of the problem include not only the economic and financial effects but also the social political and environmental and ecological effects. The Fuzzy Mathematics and AHP comprehensive evaluation method is recommended by the authors.

Two limited sets must be determined in the comprehensive evaluation. Assume the element set of evaluation is

$$U = [u_1, u_2, \ldots, u_m]$$

and the comment set is

$$V = [v_1, v_2, \ldots, v_j, \ldots, v_n]$$

the evaluation matrix of a single element can be written as

$$\underset{\sim}{R} = (r_{ij})$$

And assume the weight vector of the elements evaluated is

$$\underset{\sim}{A} = (a_1, a_2, \ldots, a_m)$$

then the fuzzy comprehensive evaluation result can be obtained in formula (2)

$$\underset{\sim}{B} = \underset{\sim}{A} * \underset{\sim}{R} = (b_1, b_2, \ldots, b_j, \ldots, b_n) \qquad \ldots \ldots (2)$$

Where u_i is elements evaluated $i = 1, 2, \ldots, m$;

v_j is comment, $j = 1, 2, \ldots, n$;

r_{ij} is degree of membership for element u_i corresponding to comment v_j; a_i is weight for each element, $a_i > 0$ and $\sum a_i = 1.0$; $m * n$ = dimension of the matrix. Here, the determination of the weight (a_i) is a very important and difficult problem as well as in the development of water resources. In this paper, the authors will review several current methods for determination of the weight and study practice for determination of weight by AHP.

II. Review of current methods for determination of the weight

There are many current methods for determination of the weight, we briefly review as follows:

1. Expert opinion method

Several experts are invited to put forward the value of the weight of each element in table form and then we can get the mean estimated value of the weight by statistics. The final mean value of the weight is feedbacked to the experts over and over again. This is a group decision making method. The reasonableness of the method depends on the quality of experts. Its merits and demerits are as follows:

(1) It is possible to collect the opinions of the experienced experts of various circle and the method is suitable for various conditions;

(2) It is a very easy and convenient method;

(3) But if there are a lot of elements in the problem, it is very difficult to determine the weight for each of them;

(4) The final mean value of the element weight is only an expected value, it can not be used directly for comparison of two projects.

2. Analytic method (α-method)

For multiobjective optimization problem,

$$\min_{x\in R} [f_1(x), f_2(x), \ldots, f_i(x), \ldots, f_p(x)] \qquad \ldots \ldots (3)$$

firstly, optimizing p single objective problem respectively

$$P_i: \quad \min_{x\in R} \quad f(x)$$

assume its optimal solution is x^j, $j=1, 2, \ldots, p$; and $f_i^j=f_i(x^j)$, $i,j=1,2,\ldots,p$; through the p points, $(f_1^j, f_2^j, \ldots, f_i^j, \ldots, f_p^j)$, $j=1,2, \ldots, p$, establishing the hyperplanes, the equations are

$$\begin{aligned} &\sum \alpha_i f_i^j = c \\ &\sum \alpha_i = 1.0 \end{aligned} \qquad \ldots \ldots (5)$$

This is a linear equations system with p+1 equations and p+1 unknowns. Then the weights α_i can be obtained by solving the linear equation system.

This method may be suitable for multiobjective and can be easily extended to the nonlinear model, but it demands that each objective be quantitatively expressed and the objective function be found. It is unsuitable to the decision making problem in which the objective can only be qualitatively expressed.

3. Method of paired comparison among objectives

In this method, the store score of each objective can be calculated by comparing each two objectives, then the store score of each objective divided respectively by the sum of the store score of all objectives and

the weight coefficient w_i for each objective is obtained.
Suppose objective set $[u_1, u_2, \ldots, u_j, \ldots, u_n]$ and let u_{ij} express the result of comparing u_i with u_j then

$$u_{ij} = \begin{cases} 1, \text{ if } u_i \text{ is more important than } u_j; \\ 0, \text{ if } u_j \text{ is more important than } u_i; \\ -, \text{ if } i=j. \end{cases}$$

and the weight is

$$w_i = \sum_{j=1}^{n} u_{ij} \Big/ \sum_{i=1}^{n} \sum_{j=1}^{n} u_{ij} \quad (i \neq j) \qquad (i=1, 2, \ldots, n) \qquad \ldots \ldots (6)$$

Though this method is simple, the relative importance between objectives is only expressed in 0 and 1. Obviously, it is so rough that the difference in relative importance between objectives can not be reflected, and if a objective is most unimportant, its weight will be zero, which is not very reasonable.

In order to improve the method mentioned above, exhaustive paired comparison technique is advanced. In this case,

$$u_{ij} = \begin{cases} 1 & \text{if } u_i \text{ is more important than } u_j; \\ 0.5 & \text{if } u_i \text{ is as same important as } u_j; \\ 0 & \text{if } u_j \text{ is more important than } u_i. \end{cases}$$

At the same time, a virtual objective u_{n+1} is introduced into the objective set to avoid the weight of some objectives being zero, and let u_{n+1} be the least important in the set, and then the new objective set can be expressed as $[u_1, u_2, \ldots, u_n, u_{n+1}]$, $\quad u_{i,n+1} = 1, \quad i = 1, 2, \ldots, n$.
Finaly, the weight for each objective can be got in equation (7)

$$w_i = \sum_{j=1}^{n+1} u_{ij} \Big/ \sum_{i=1}^{n+1} \sum_{j=1}^{n+1} u_{ij} \quad (i \neq j) \qquad \ldots \ldots (7)$$

Where the weight w_{n+1} for the virtual objective is equal to zero, i.e., $w_{n+1}=0$. Although some improvement on the second method is made in this method, there still exsits the demerit that the comparing result is too simple.

4. Circle making method

In the circle marking method, firstly determine the rate of importance between each two objectives by successively comparing one objective with its next from the first one to the last, and then calculate the rate among all objectives by comparing in the same way described above from the last one to the first. According to the rate, the weights w_i for each objective can finally be calculated.

for example, suppose the objective set of a system is Z_i (i=1,2,3,4,5,6), the process to determine the weight for the objectives Z_i is shown in table (1)

Table (1)

Objectives	z_1	z_2	z_3	z_4	z_5	z_6
the rate of importance between the objectives	2	1	3	5	.25	1
the rate of importance among the objectives	7.5	3.75	3.75	1.25	.25	1.0
the weights w (%)	42.86	21.43	21.43	7.14	1.43	5.71

This method is simple and convenient. But the comparison is based on the final objective, so the comparability and transitivity between each pair of the objectives are demanded, and the method does not deeply concern how to get the rate between qualitative objectives and quantitative objectives with different dimension.

5. Binomial coefficient method

When the method is applied, the objectives need to be nondimensionized and standardized firstly, and then a qualitative objective has to be changed into a quantitative by marking. The all objectives can be standardized in formula (8)

$$r_{ij} = (f_{ij} - \min f_{ij}) / \max (f_{ij} - \min f_{ij}) \qquad i = 1,\dots,m;\ j = 1,\dots,n \qquad \dots\dots(8)$$

where f_{ij} is a value of objective i.

According to the importance of each objective, the objectives are arranged in order. When $m \longrightarrow \infty$, the probability distribution for the m objectives is similarly subordinated to normal distribution, and then the weights for each objective w_i are calculated in formula (9).

$$w_i = C_m^{i-1}/2^m, \quad (i=1,\ 2,\ \dots,\ m) \qquad \dots\ \dots\ (9)$$

where $\sum_{i=1}^{m} w_i = 1.0$

The method makes a qualitative objective nondimensionized and standardized and establishes the calculating formula for w_i. All this is an improvement on the third method described above. But marking a qualitative objective and determining the priority of the objectives are the problems which remain to be deeply studied. In addition to the methods above mentioned, there are some others for determining the weight. Here we shall not describe them one by one.

To sum up, the current methods for determing the weight have the following demerits:

1) The comparison and judgement among the objectives are not perfect;
2) The comparison result being simpler, and lacking suitable scalar, it is difficult to reflect the difference in importance among the objectives.
3) The calculation of weights is simple, lacking profound mathematical basis.
4) It lacks effective method for change a qualitative objective into quantitative objective.
5) It is not convenient to check the consistency of decision maker's thinking process.

AHP can just overcome the demerits mentioned above and has become a practical and simple decision making method. The following practical examples will further prove that AHP is an effective method for determining the weight.

III. Application of AHP in determination of the weight for development of water resources

1. Index system of comprehensive evaluation and hierarchy model

According to the character and practice for development of water resources in China, we establish three different models to determine the weights.

(1) Model for comprehensive evaluation of water conservancy project. The Structure is composed of three levels. The first level is general object. The second level is criteria, it contains the following three aspects: economic and financial, social and political, and environmental and ecological effectiveness. The third level is indexes, it contains 12 elements.

(2) Model for comprehensive evaluation of operation and management of water conservancy project. The hierarchy structure is composed of four levels: The first is general object; The second is criterion level, it contains the following four aspects: productive management, feasibility of project goal, comprehensive utilization of water and soil resources and the management effectiveness; The third is main index level, it contains 20 elements; The fourth level is influence element level, it contains 21 elements.

(3) Model for optimum allocation of water resources for an economic region.

According to the character of certain economic region in China we select the maximumof social and economic effectiveness of an economic region as a general objective, then we transform it into three objectives: maximum output value of agriculture, maximum output value of industry and minimum cost for supplying water. The influential factors are divided into political, economic, technical, environmental and social effectiveness, which contain eighteen elements (indexes). The interdependency among the elements at the second level is considered in the model.

2.Establishment of evaluation matrix for single element

The indexes in hierarchy structure mentioned above can be divided into two classes quantitative index and qualitative index according to the character.

(1) Evaluation of single element for a quantitative index.

Consulting relative standards, stipulations and the statistic data of the different departments in the different regions, the reasonable scope of the index [a,b] is selected, the membership function is established in fuzzy mathematics, taking the value of the index into the function, and then the degree of membership r_{ij} can be got and the judging result is obtained.

$$\underset{\sim}{R}_i = (r_{i1}, r_{i2}, \ldots, r_{in}), \ i = 1,2,\ldots,m$$

(2) Evaluation of single element for a qualitative index.

Generally, the experts are invited to mark the index, then comprehensive value is adopted, We suggest that the each index be diveded into five classes according to influence degree of the index and consulting comment set, then the evaluation result is got by comparing the index with the different classees and determining the corresponding degree of membership.

Fuzzy mathematics method may also be adopted for evaluation of the qualitative index (for example: inundation loss). Firstly the highest and lowest values need to be selected (expresssing by corresponding stage or inundated area, population of inundation), and degree of membership $0(v_n)$ and $1(v_1)$ are assumed, then the function of membership is established by dividing the scope between v_1 and v_n into several classes. And then by putting the concrete value of the indexes into the corresponding membership function, evaluation results can be obtained.

Summing up the evaluation result to the quantitative and qualitative index, we can obtain the evaluation matrix for single element.

$$\underset{\sim}{R} = \begin{bmatrix} \underset{\sim}{R}_1 \\ \underset{\sim}{R}_2 \\ \vdots \\ \cdot \\ \underset{\sim}{R}_m \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & & \vdots \\ \cdots & \cdots & \cdots & \cdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{bmatrix}$$

3. Determination of weight for each element by AHP

In order to understand easily, take the model for comprehensive evaluation of operation and management of water conservancy projects for example, the model consists of four levels, in which there are one objective, four criteria twenty and twenty-one elements (indexes). In judgment, the comment set $V = [v_1, v_2, v_3, v_4, v_5]$ respectively corresponds to five classes of "very good" , "good", "general", "bad", "very bad".
Adopting the saclar of 1-9, the matrixes are established by pairwise comparison of elements at each level from bottom to top. Then the weight for each element of each level are calculated according to the matrixes.

4. Multilevel comprehensive evaluation by fuzzy mathematics and AHP

Making use of the evaluation matrixes for single element of each level and corresponding weights, the composition of fuzzy relation is made from bottom to top of the corresponding hierarchy structure. Finally, the comprehensive evaluation result $\underset{\sim}{B}$ to the total objective is obtained, i.e:

$$\underset{\sim}{B} = \underset{\sim}{A} \cdot \underset{\sim}{R} = (b_1, b_2, b_3, b_4, b_5)$$

In this example, the final evaluation result is

$$\underset{\sim}{B} = \underset{\sim}{A} \cdot \underset{\sim}{R} = (0.473, 0.284, 0.13, 0.05, 0.063)$$

Where: $b_{max} = b_1 = 0.473$, then the comment v_1 (very good) corresponding to b_{max} or the degree of membership to v_1 and v_2 being 47.3 percent and 28.4 percent respectively, it is thought to be the comprehensive evaluation result for the water conservancy project.
To the model for optimum allocation of economic regional water resources, the " group discussing " method for giving a judgement matrix is discussed and adopted. By calculating, the weights corresponding to three objectives, the agricultural output value, the industry output value and the cost for supplying water are 0.43986, 0.43249, 0.12765 respectively. After analyzing by relevant personnel and comparison of the real conditions of the economic region, the weights are thought to be reasonable.

IV. Discussing on some questions

1. "Group discussing" method for giving a judgment matrix

In application process, we advance group discussing method for giving a judgment matrix on the basis of system thinking and the complementation principle. It is to say that every one's knowledge is limited, every one's mode of thinking and favour are different, so no one can perfectly understand a very complicated problem. But if we put people's knowledge, mode of thinking and favour together organically, we can more perfectly and accurately understand the problem.

The practice shows that the method can directly give a judgment matrix. There are the following advantages over the group AHP.
1) Saving a lot of work.
2) Avoiding the difficulty in treating many judgment values for same comparison.
3) Overcoming the demerits of group AHP in which a person can not understand the problem perfectly. Therefore, we suggest that the group discussing method be adopted in solving complicated problem.

2. On the consistency in AHP

There exist two formulas in book [1] and book [2] . They are the following:

$$C.R._k = \sum_{i=1}^{m} a_i C.I._k^{(i)} \Big/ \sum_{i=1}^{m} a_i R.I._k^{(i)} \qquad ....(10)$$

$$C.R._k = (C.R._{k-1} + C.I._k / R.I._k) \qquad ....(11)$$

where, for the meaning of each symbol see [1] and [2]. Firstly,

$$C.R._k = \sum_{i=1}^{m} a_i (C.R._i R.I._i) \Big/ \sum_{i=1}^{m} a_i R.I._i = \sum_{i=1}^{m} (a_i R.I._i) C.R. \Big/ \sum_{i=1}^{m} a_i R.I._i$$

In the formula, if $C.R._i < 0.1$ ($i=1,2,\dots,m$), $C.R._k < 0.1$.
This means that if the consistency of each matrix can be accepted. It is unnecessary to check the consistency in formula (10).

Secondly, we calculate the consistency of a certain existing application example in formula (11), making use of the computer program made by us. The results show the consistency ratio of part example is greater than 0.1. It is obvious. Suppose $C.R._{k-1}$ equals critical value 0.1, then $C.R._{k-1} = (0.1 + C.I._k / R.I._k)$ greater than 0.1 as long as $C.I._k / R.I._k$ is not equal to zero. We think that the improvement on formula (11) can be made from two hands. The first is that different standards should be adopted for different levels. The second is that the influnce of the location of the level in the structure and the total number of judgment matrixes should be considered. It is a pity that we can not do more concerete work because of lack of time.

REFERENCES

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