

## SUBJECTIVE AND QUANTITATIVE ANALYSIS OF FAULT TREE BY USE OF AHP

Y.Kameyama\*, H.Sayama\*, K.Suzuki\*, K.Okemoto\*\*  
and H.Nakayama\*\*\*

\* Faculty of Engineering, Okayama University, Okayama 700, Japan

\*\* Kubota Ironwork Co.Ltd., Osaka 556, Japan

\*\*\* Faculty of Science, Konan University, Kobe 658, Japan

### ABSTRACT

A procedure is proposed which can subjectively estimate the relative occurrence probability of each fundamental events in fault tree by applying the pairwise comparison in AHP. A questionnaire survey is presented for the relation between every modifier which explains the result of the pairwise comparison by word, and sensuous numerical values which correspond to the modifier. From the result of opinionnaire, the representative numerical value is assigned for every modifier. The proposed procedure could be applied to the quantitative analysis of fault tree for the prevention of a dust explosion or fire in the grain elevator. It is possible to estimate the effect of the variation in the occurrence probability of each fundamental events on the variation in occurrence probability of top event, even if some of data would be lacking.

### INTRODUCTION

Fault tree analysis, FTA, is one of the effective technique to examine the safety and reliability of systems (Henley E. J. et.al. 1981). There are qualitative and quantitative analysis in FTA. When we want to quantitatively assess the occurrence probability of top event, the data of occurrence probability for every fundamental events in fault tree are necessary in addition to the structure of tree. But it is not easy to collect the data of probability in the tree. In this work, a procedure is proposed which can subjectively estimate the probability in fault tree. The procedure utilizes the sensuous knowledge for the relative occurrence probabilities which have been accumulated by the experienced operators and experts in the industry. The method is applied to the safety assessment of grain processing an handling facilities.

### QUANTITATIVE FAULT TREE ANALYSIS

If we construct a fault tree where the top event is system failure and the fundamental events are component failures with statistical independence,  $g_{top}$ , the probability of top event in the fault tree is expressed by

$$g_{top} = g(q) \quad q = (q_1, q_2, \dots, q_n) \quad (1)$$

where  $q_i$  is the probability of fundamental event  $X_i$  ( $i=1, \dots, n$ ).

The importance of  $X_i$  is defined as the rate at which probability of top event increases as the probability of fundamental event increases (Birnbaum 1969) by

$$I_g(i) = \frac{\partial g(q)}{\partial q_i}, \quad (i=1, \dots, n). \quad (2)$$

CIg(i), the criticality importance of  $X_i$ , is defined as the rate at which percentage variation concerned with probability of top event increases as the percentage variation concerned with probability of fundamental event increases by

$$CIg(i) = \frac{q_i}{g_{top}} \cdot \frac{\partial g(q)}{\partial q_i}, \quad (i=1, \dots, n) \quad (3)$$

#### NUMERICAL EXPRESSION OF RESULT ON PAIRWISE COMPARISON

When the occurrence probabilities of fundamental event  $X_i$  and  $X_j$  are represented as  $q_i$  and  $q_j$  respectively, the result of pairwise comparison between  $q_i$  and  $q_j$ ,  $a_{ij}=q_i/q_j$  is called relative occurrence probability of fundamental event  $X_i$  and  $X_j$ . If such the relation as  $a_{ik}=a_{ij} a_{jk}$  in Figure 1 is realized, we call that the relation satisfies cardinal consistency among the results of pairwise comparison  $a_{ik}$  (Saaty T. L. 1980). Judging from our feeling, the relations expressed in Table 1 are valid in the synthesis of scales (modifiers) which represent the result of pairwise comparison.

We questioned to ten persons who had the experience in safety and risk analysis. The contents of question are the relation between the modifier (or scale) which explains the result of pairwise comparison by word and sensuous numerical values which corresponds to the modifier. Figure 2 indicates a part of result on the questionnaire. From our questionnaire result and its numerical consistency (Kameyama Y. et.al. 1987), we propose the representative numerical value for each scale (modifier) in Table 2.

#### PROCEDURE FOR SUBJECTIVE FAULT TREE ANALYSIS

Figure 3 explains the procedure for subjective estimation of the occurrence probability of each fundamental event. In this procedure, the result of each pairwise comparison for the occurrence probability by word is exchanged for the representative numerical value in Table 2. For the pairwise comparison matrix, the consistency index of 0.10 or less is also considered acceptable. Figure 3 indicates also the procedure for estimation of importance and criticality importance of each fundamental event for the top event in the fault tree.

#### BUCKET ELEVATOR AND ITS FAULT TREE

Figure 4 exhibits the schematic of a grain elevator that contains two main sections, namely the storage bins and the work house. Figure 5 indicates the bucket elevator which has the high risk on dust explosion or fire in the grain elevator (Lai F.S. et.al. 1984). Figure 6 shows the fault tree for a dust explosion or fire in the bucket elevator where only the ignition source is analyzed in detail. This fault tree was primarily constructed by us, and then partly modified by Prof.H.Tanaka et.al. The intermediate events and fundamental events are explained in Table 3.

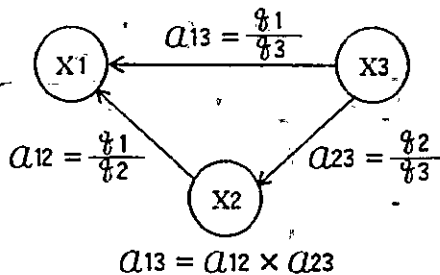


Figure 1. Consistency among results of pairwise-comparison

Table 1. Synthesis of scales (modifiers) for representing the result of pairwise-comparison

$a_{ij}$	$a_{jk}$	$a_{ij} \times a_{jk}$
Equally Weakly more Strongly more Demonstrably more Absolutely more	Equally Weakly more Strongly more Demonstrably more Absolutely more	Equally Strongly more Demonstrably more Absolutely more Absolutely more

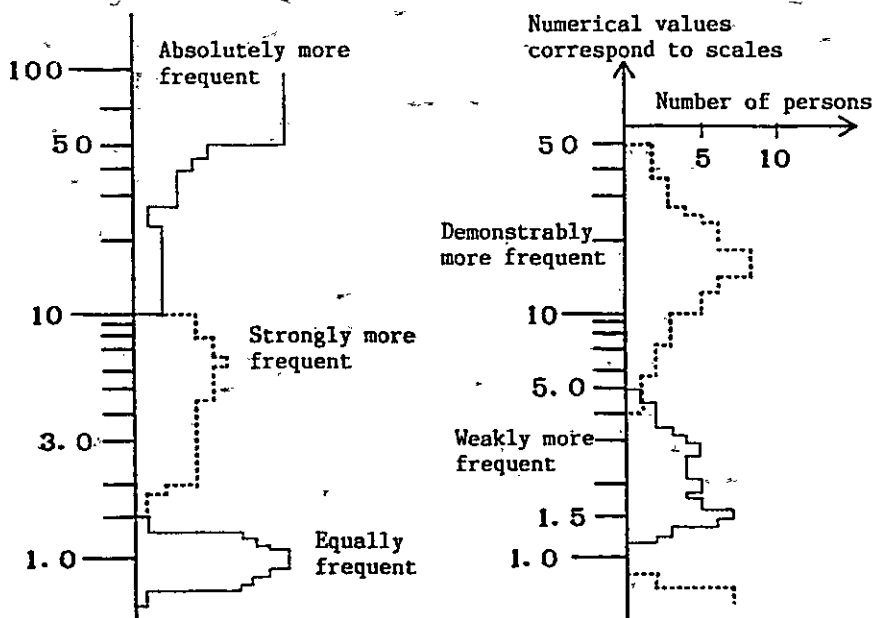


Figure 2. Result of questionnaire for the relation between comparison scale (modifier) and interval of numerical value which is intuitively felt to correspond to the scale

Table 2. Our proposal for representative numerical values which correspond to comparison scales explained by words

scales explained by words	Saaty's values	Our values
Equally frequent	1	1
Weakly more frequent	3	1.7
Strongly more frequent	5	3
Demonstrately more frequent	7	10
Absolutely more frequent	9	100

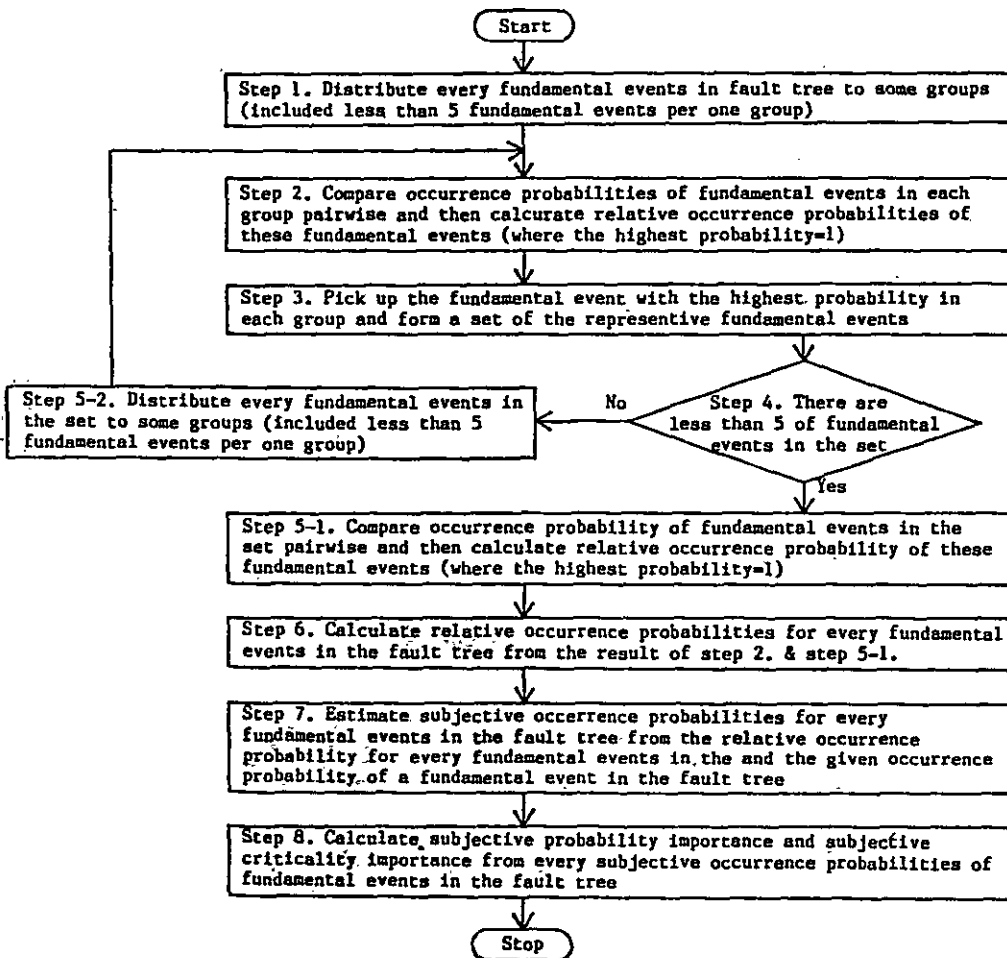


Figure 3. Procedure for subjective & quantitative analysis of fault tree

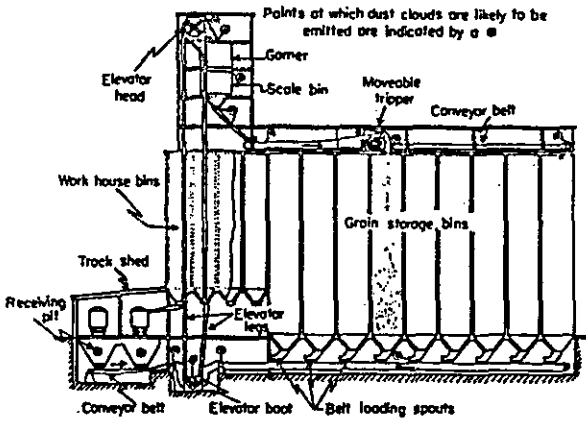


Figure 4. Schematic of a typical elevator

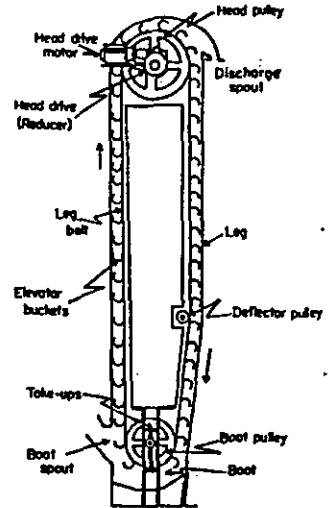


Figure 5. Bucket elevator section of typical grain handling facility

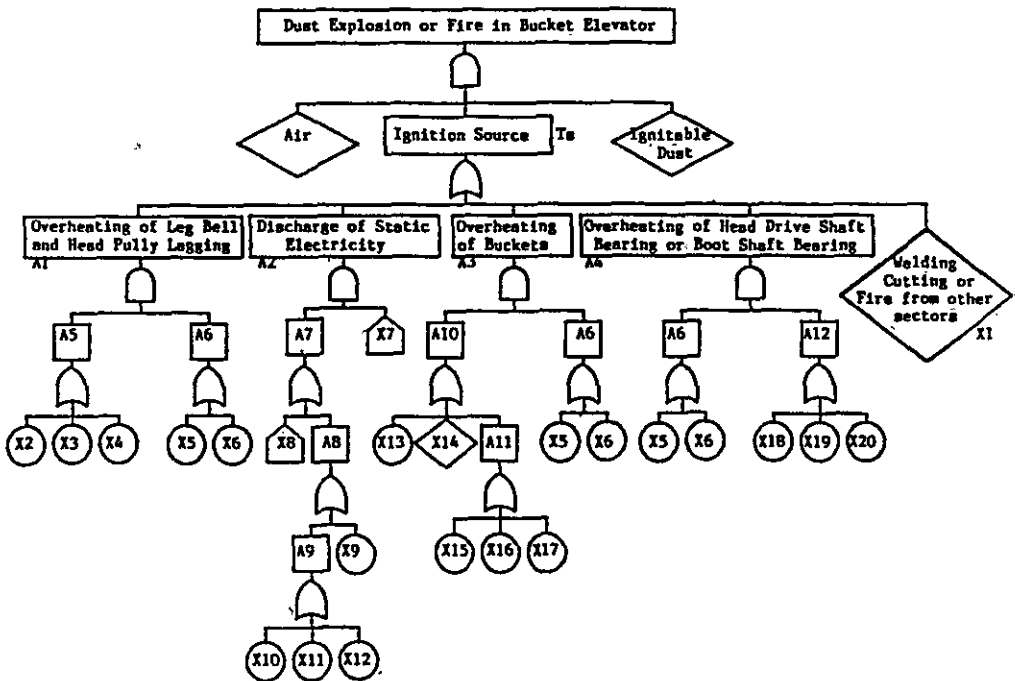


Figure 6. Fault tree for a dust explosion or fire in the bucket elevator section

Table 3. Intermediate & fundamental events in the fault tree

INTERMEDIATE EVENTS	
A1	Overheating of the belt and head pulley lagging
A2	Discharge of static electricity
A3	Overheating of the buckets
A4	Overheating of the head drive shaft bearing or boot shaft bearing
A5	Slippage between the leg belt and head pulley
A6	Continuation of operation of the head drive motor
A7	Accumulation of static electricity in the leg
A8	Inadequate leakage of static electricity from the leg
A9	Poor grounding of the head shaft, boot shaft or deflector shaft
A10	Friction between the inside walls of the leg and buckets
A11	Low tension of the leg belt
A12	Malfunction of the head drive shaft bearing or boot shaft bearings
FUNDAMENTAL EVENTS	
X1	Welding, cutting or fire from other sectors
X2	Maladjustment of the boot shaft take-ups
X3	Wear of the head pulley lagging
X4	Overloading of the leg
X5	Failure of the head drive motor to stop due to failure of magnetic starter
X6	Operator failure in stopping the head drive motor
X7	Presence of the spark gap
X8	Generation of static electricity in the leg
X9	Low humidity in the leg
X10	Use of nonconductive lubricant
X11	Breaking of the grounding wire
X12	Improper installation of the grounding wire
X13	Malfunction of the buckets
X14	Foreign materials in the leg
X15	Malfunction of the leg belt
X16	Malfunction of the boot shaft take-ups
X17	Maladjustment of the boot shaft take-ups
X18	Inadequate lubrication
X19	Failure of bearings
X20	Overload of bearings

Table 4. Subjective criticality importance of the fundamental events to the top event in the fault tree

Fundamental events	Relative frequency for occurrence of fundamental event	Subjective frequency for occurrence of fundamental event	Subjective importance of fundamental event	priority	Subjective criticality importance of fundamental event	priority
X1	0.1363	0.0409	0.8871	1	0.2432	2
X2	0.2360	0.0708	0.0831	10	0.0395	10
X3	0.3573	0.1072	0.0865	9	0.0622	8
X4	0.4670	0.1041	0.0898	6	0.0844	5
X5	0.1026	0.0308	0.7442	3	0.1534	3
X6	0.3126	0.0949	0.7969	2	0.5070	1
X7	0.1026	0.0308	0.0938	5	0.0194	12
X8	1.0000	0.3001	0.0096	6	0.0194	12
X9	0.7123	0.2138	0.0064	17	0.0091	17
X10	0.0618	0.0185	0.0051	19	0.0006	19
X11	0.3451	0.1636	0.0060	18	0.0065	18
X12	0.0618	0.0185	0.0051	20	0.0006	19
X13	0.0794	0.0238	0.0734	13	0.0117	14
X14	0.7939	0.2383	0.0940	4	0.1502	4
X15	0.0794	0.0238	0.0734	13	0.0117	14
X16	0.0794	0.0238	0.0734	13	0.0117	14
X17	0.2450	0.0735	0.0773	12	0.0381	11
X18	0.4165	0.1250	0.0869	7	0.0728	6
X19	0.2450	0.0735	0.0830	11	0.0472	9
X20	0.4165	0.0125	0.0869	7	0.0728	6

## RESULTS AND DISCUSSION

In this section, the results of a subjective and quantitative fault tree analysis of the elevator are presented and discussed. In order to calculate the subjective and relative probability of fundamental events, twenty fundamental events in the fault tree have been divided to five groups with four events. From the result of pairwise comparison in each group, the event with the highest relative probability is  $X_4, X_8, X_9, X_{14}$  and  $X_{20}$  respectively. These events become a new set of fundamental events.  $X_8$  is the event with the highest probability in the new set. Every relative occurrence probability of fundamental events in the fault tree based on the probability of  $X_8$  is exhibited in Table 4. It is assumed that the occurrence probability of  $X_{18}$ , namely inadequate lubrication is known to be  $q_8=0.125$ . In other words, the erroneous use of lubrication oil happened once per 8 years. As the relative occurrence probability of  $X_8$  is 0.4165 in Table 4, occurrence probability of every fundamental events is subjectively estimated by multiplying 0.3001 ( $0.125/0.4165$ ) to every relative probability of them and is shown also in Table 4.

Subjective importance and subjective criticality importance of every fundamental event could be calculated from the subjective occurrence probability of every fundamental event and are also exhibited in Table 4. The magnitude of subjective criticality importance could indicate the priority of action required for safety of system. For example, it is very effective for the prevention of dust explosion or fire in the bucket elevator to prevent such the fundamental events as  $X_6$ , namely operator failure to stop the head drive motor.

## CONCLUSIONS

For the quantitative fault tree analysis, it is not easy to collect the data of probability for every fundamental events. In this work, a procedure was proposed which could quantify the sensuous knowledge with relative occurrence probability of each events in fault tree by applying the pairwise comparison matrix in AHP. Even if some of data for occurrence probability of each event would be lacking, it is possible to estimate the effect of the variation in occurrence probability of each fundamental event on the variation in occurrence probability of top event. The procedure was applied to the safety analysis for prevention of a dust explosion or fire in the grain elevator and could determine the priority of action required to keep the system safe.

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