USING ANP APPROACH
FOR NETWORK REVENUE MANAGEMENT PROBLEMS

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ABSTRACT

Revenue management (RM) is the process of understanding, anticipating and influencing consumer behavior in order to maximize revenue. The challenge is to sell the right resources to the right customer at the right time for the right price through the right channel. Network revenue management models attempt to maximize revenue when customers buy bundles of multiple resources. An Analytic Network Process (ANP)-based framework for RM problems structuring and combining specific methods is presented. RM addresses three basic categories of demand-management decisions: price, quantity, and structural decisions. Specific models are used to model and to solve basic RM decisions. Combinations of the solutions are given by sub-networks in an ANP model.

Keywords: Revenue management, multicriteria decisions, price decisions, quantity decisions, structural decisions, Analytic network process, Dynamic Network Process

1. Introduction

Revenue management (RM) is the process of understanding, anticipating and influencing consumer behavior in order to maximize revenue or profits from fixed, perishable resources. Recent years have seen great successes of revenue management, notably in the airline, hotel, and car rental business. Currently, an increasing number of industries is exploring to adopt similar concepts (see Talluri, van Ryzin, 2004). What is new about RM is not the demand-management decisions themselves but rather how these decisions are made. The true innovation of RM lies in the method of decision making.

Revenue Management is to sell the right product, to the right customer at the right time, for the right price through the right channel by maximizing revenue. RM is the art and science of predicting real-time customer demand and optimizing the price and availability of products according to the demand. RM addresses three basic categories of demand-management decisions: structural, price, and quantity decisions. Network revenue management models attempt to maximize revenue when customers buy bundles of multiple resources. The dependence among the resources in such cases is created by customer demand.

For the basic specific problems are proposed many appropriate methods (see Talluri, van Ryzin, 2004). An Analytic Network Process (ANP) - based framework for RM problems structuring and combining specific methods is presented in this paper. Combinations of the solutions are given by subnetworks in an ANP model. RM problems are complex dynamic problems. The DNP (Dynamic Network Process) method was used for dynamic extensions.

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2. ANP structure of the problem

The Analytic Hierarchy Process (AHP) is the method for setting priorities (Saaty, 1996). A priority scale based on reference is the AHP way to standardize non-unique scales in order to combine multiple performance measures. The AHP derives ratio scale priorities by making paired comparisons of elements on a common hierarchy level by using a 1 to 9 scale of absolute numbers. The absolute number from the scale is an approximation to the ratio w_j / w_k and then is possible to derive values of w_j and w_k. The AHP method uses the general model for synthesis of the performance measures in the hierarchical structure.
The Analytic Network Process (ANP) is the method (Saaty, 2001) that makes it possible to deal systematically with all kinds of dependence and feedback in the performance system. The structure of the ANP model is described by clusters of elements connected by their dependence on one another. A cluster groups elements that share a set of attributes. At least one element in each of these clusters is connected to some element in another cluster. These connections indicate the flow of influence between the elements.
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RM problems are complex dynamic problems. The ANP has been static but for RM problem is very important time dependent decision making. The DNP (Dynamic Network Process) method was introduced (Saaty, 2003). There are two ways to study dynamic decisions: structural, by including scenarios, and functional by explicitly involving time in the judgment process. For the functional dynamics there are analytic or numerical solutions. The basic idea with the numerical approach is to obtain the time dependent principal eigenvector by simulation. The Dynamic Network Process seems to be the appropriate instrument for analyzing dynamic networks (Fiala, 2006). The method is appropriate also for the specific features of RM problems. The method computes time dependent weights for decisions and combinations of decisions. We used the ANP software Super Decisions developed by Creative Decisions Foundation (CDF) for some experiments for testing the possibilities of the expression and evaluation of the dynamic RM models. Decisions can be two possibilities - accept or reject a request for a product. The clusters in RM problem can be resources, customers, time, prices, channels, and decisions (see Figure 1).

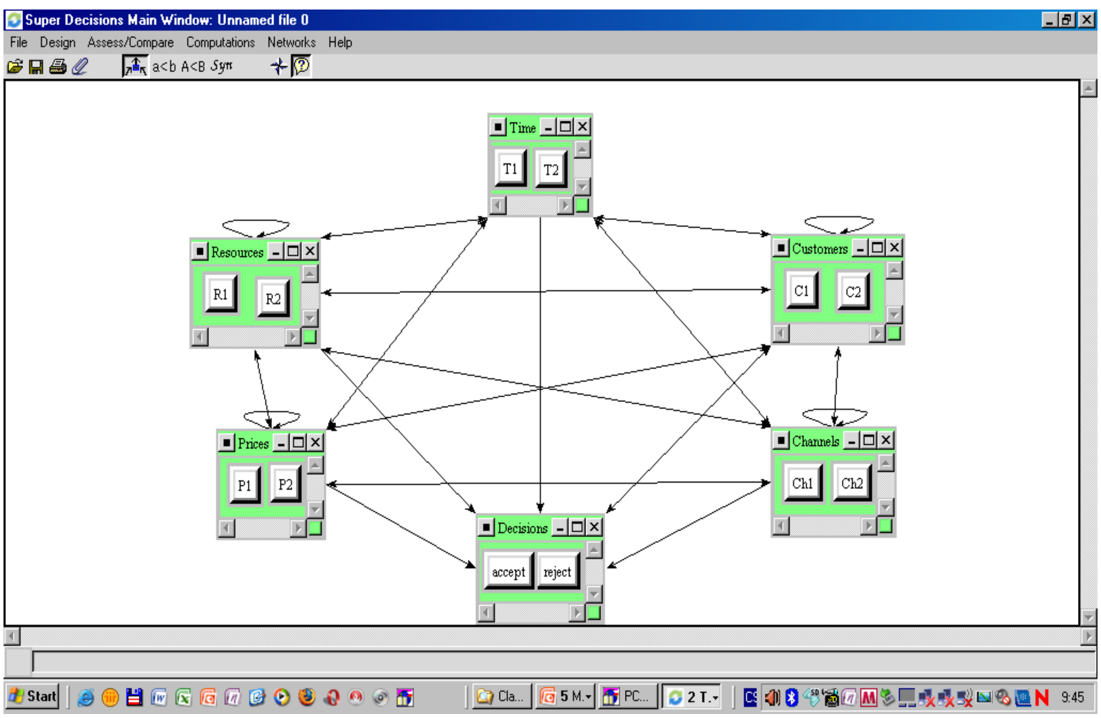

Figure 1 Super Decisions - RM model

3. Sub-networks

The basic ANP model is completed by specific sub-networks. The sub-networks are used to model important features of the RM problems. The most important features in our ANP-based framework for revenue management are captured in sub-networks:

- time dependent resources,
- products,
- network revenue management,
- price-quantity-structure network.

Time dependent resources

A specific sub-network is devoted to model time dependent amounts of resources. The time dependent amount of resources is given by previous decisions. The sub-network connects clusters: time, resources and decisions.

Products

A product is a sub-collection of the available resources. An (m, n) matrix $A = [a_{ij}]$ is defined such that a_{ij} represents the amount of resource i used to produce one unit of product j. Every column j of A represents a different product and the collection $M = \{A_{.1}, \dots, A_{.n}\}$ is the menu of products offered by the seller.

Network revenue management

The quantity-based revenue management of multiple resources is referred as network revenue management. This class of problems arises for example in airline, hotel, and railway management. In the airline case, the problem is managing capacities of a set of connecting flights across a network, so called a hub-and-spoke network (see Figure 2). In the hotel case, the problem is managing room capacity on consecutive days when customers stay multiple nights.

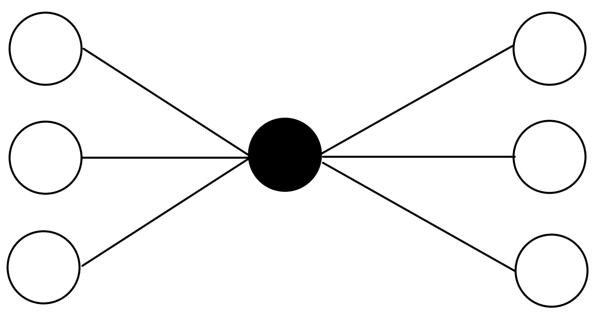

Figure 2 Hub-and-spoke network

The basic model of the network revenue management problem can be formulated as follows (see Talluri, van Ryzin, 2004): The network has m resources which can be used to provide n products. We define the incidence matrix $A = [a_{ij}]$, $i = 1, 2, \dots, m$, $j = 1, 2, \dots, n$, where

$a_{ij} = 1$, if resource i is used by product j, and
$a_{ij} = 0$, otherwise.

The j-th column of A, denoted a_j, is the incidence vector for product j. The notation $i \in a_j$ indicates that resource i is used by product j.

The state of the network is described by a vector $x = (x_1, x_2, \dots, x_m)$ of resource capacities. If product j is sold, the state of the network changes to $x - a_j$.

Time is discrete, there are T periods and the index t represents the current time, $t = 1, 2, \ldots, T$. Assuming within each time period t at most one request for a product can arrive.

Price-quantity-structure network
RM addresses three basic categories of demand-management decisions:

- Price decisions:
- Quantity decisions:
- Structural decisions:

The price-quantity-structure network is given by interdependences of the three very important factors. The solutions of three basic categories of demand-management decisions are solved by basic methods described in next paragraphs. Interdependences are modeled and analyzed in the ANP sub-network.

4. Price decisions

The basic pricing model of the network revenue management problem is formulated as a stochastic dynamic programming problem whose exact solution is computationally intractable. Most approximation methods are based on one of two basic approaches: to use a simplified network model or to decompose the network problem into a collection of single-resource problems.

The deterministic models assume that the seller has perfect information about the demand process. Deterministic models are easy to analyze and they provide a good approximation for the more realistic yet complicated stochastic models. Deterministic solutions are in some cases asymptotically optimal for the stochastic demand problem (Cooper, 2002).

The simplest deterministic model considers the case of a monopolist selling a single product to a price sensitive demand during a period $[0, T]$. The initial inventory is C_0, demand is deterministic with time dependent and price sensitive intensity $\mu(p, t)$. The instantaneous revenue function $r(p, t) = p\mu(p, t)$ is assumed to be concave as in most real situations. The general revenue management problem can be written in this case as follows:

$$\max_{P} \int_{0}^{T} p_t \mu(p_t, t) dt \tag{1}$$

subject to

$$\int_{0}^{T} \mu(p_t, t) dt \leq C_0 . \tag{2}$$

This is a standard problem in calculus of variations. The optimality condition is given by

$$p_t^* = \lambda - \frac{\mu(p_t^*, t)}{\mu_p(p_t^*, t)}, \tag{3}$$

where λ is the Lagrangian multiplier for the constraint, μ_p is the partial derivative of μ with respect to the price.

5. Quantity decisions

The maximum expected revenue, given remaining capacity x in time period t, must satisfy the Bellman equation. The equation cannot be solved exactly for most networks of realistic size. Solutions are based on approximations of various types. There are two important criteria when judging network approximation methods: accuracy and speed. Among the most useful information provided by an approximation method are estimates of bid prices (see Talluri, van Ryzin, 2004).

We introduce Deterministic Linear Programming (DLP) method. The approach is to use a simplified network model, for example posing the problem as a static mathematical program.
The DLP method uses the approximation

$$V_t^{LP}(x) = \max r^T y \tag{4}$$

subject to

$$Ay \le x \tag{5}$$
$$0 \le y \le E[D] \tag{6}$$

where $D = (D_1, D_2, \ldots, D_n)$ is the vector of demand over the periods $t, t+1, \ldots, T$, for product j, $j = 1, 2, \ldots, n$, and $r = (r_1, r_2, \ldots, r_n)$ is the vector of revenues associated with the n products. The decision vector $y = (y_1, y_2, \ldots, y_n)$ represents partitioned allocation of capacity for each of the n products. The approximation effectively treats demand as if it were deterministic and equal to its mean $E[D]$.

The optimal dual variables, π^{LP}, associated with the constraints $Ay \le x$, are used as bid prices. The DLP was among the first models analyzed for network RM (see Talluri, van Ryzin, 2004). The main advantage of the DLP model is that it is computationally very efficient to solve. Due to its simplicity and speed, it is a popular in practice. The weakness of the DLP approximation is that it considers only the mean demand and ignores all other distributional information. The performance of the DLP method depends on the type of network, the order in which fare products arrive and the frequency of re-optimization.

6. Structural decisions

One of structural decisions is how to bundle products. Auctions are important market mechanisms for the allocation of goods and services. Combinatorial auctions are those auctions in which bidders can place bids on combinations of items, so called bundles. The advantage of combinatorial auctions is that the bidder can more fully express his preferences. This is particular important when items are complements. The auction designer also derives value from combinatorial auctions. Allowing bidders more fully to express preferences often leads to improved economic efficiency and greater auction revenues. However, alongside their advantages, combinatorial auctions raise a host of questions and challenges (see Cramton et al., 2006, de Vries and Vohra, 2003).

The problem, called the winner determination problem, has received considerable attention in the literature. The problem is formulated as: Given a set of bids in a combinatorial auction, find an allocation of items to bidders that maximizes the seller's revenue. It was introduced many important ideas, such as the mathematical programming formulation of the winner determination problem, the connection between the winner determination problem and the set packing problem as well as the issue of complexity. The The ANP approach is possible to be applied for problems of combinatorial auctions (see Fiala, 2009).

7. Conclusions

RM problems are the important subjects of an intensive economic research. A possible flexible ANP/DNP framework is presented. Analytic Network Process methodology is useful for structuring the RM problem and for combining specific models. Sub-networks are used for sophisticated analyses of RM processes. Specific models are used to model and to solve basic RM decisions (price, quantity, structure). Approximations, heuristics, or iterative approaches are used for solving the specific models. Dynamic Network Process is an appropriate approach for explicitly involving time in the RM processes. The combination of such approaches can give more complex views on RM problem.

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