THE GREY STRATEGY FOR A WATER RESOURCE ALLOCATION USING AHP

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ABSTRACT

The Analytic Hierarchy Process (AHP) is an effective decision analysis method to manipulate the multiobjective, multicriteria and multihierarchy complex system. To build the pairwise comparison matrix A for each criteria is a main step using the Saaty's procedure. A real socio-economic system includes simultaneously both the non-quantitative factor and a lot of imprecise statistical data. We consider this case as a grey system hierarchical analysis problem.

Much of classical hierarchy analysis may be accomplished with comparison matrix by expert. In this paper, let A be the n*n matrix which entries are a_{ij}. We combine the strategy method of the grey system theory [3] with AHP to develop hierarchical analysis. The grey analytic hierarchical process method can be used to aid the expert's judgement to supply ratioes a_{ij}. When we compute the entries, the ecology benefit, the social benefit and economic benefit are considered. The method has been tested in Liaoning province to study the optimum water supply allocation during 1987-2000 years.

1. INTRODUCTION

The Liao-He river basin, which is located in the middle area of Liaoning province of China, is an important industrial and agricutural base. In this area, there are many large cities and irrigation systems.

The great demand and the shortage of water supply are two critical problems in this region. It is necessary to solve the problem, that is, to fully and appropriately plan the utilization of the limited water resource

The planning of a water supply system is a complex system engineering problem. It is affected not only by the location climate and resource constraint but also by political factor and social environment.

The linear programming method is a very effective method in application of the development planning of water supply systems. In this method the features of the location and climate conditions have influence on the selection of the decision variables and constraints. The objective function depends upon the economical condition, the political factor and the social environment. Therefore, the coefficients of the objective function denote the comprehensive benefit per unit volume water because the decision variable is denoted as the intensity of water flowing between water source and users in water allocation planning. We must describe the objective function by the political, economical and the public benefit. Therefore, how to calculate the comprehensive benefit per unit volume water becomes the key of the problem. Then, the planning of water supply system is able to be transformed into the linear programming problem.

In this paper, we combine the grey system theory with the analytic hierarchy process to present an approach of the grey system AHP. This method has been tested in Liaoning province to study the comprehensive benefit per unit volume water.

2. THE HIERARCHICAL STRUCTURE OF THE OBJECTIVE FUNCTION IN WATER ALLOCATION PLANNING FOR LIAO-HE BASIN

The Liao-He river flows through ten cities and their suburb agricultural area. We select underground and fresh surface water as water source and consider to whom the amount of water is supplied to the industries, residential areas and irrigation systems in each city as water users. Their sub-objective funtions denote the comprehensive benefits as follows:

(1) Sub-objective 1: increasing the gross value of industrial and agricultural production

Water is an important kind of material in industrial and agricultural production. Each water user has the different priority and water expansion norm under various producted condition. We select the gross value per unit volume water to describe the effect of this sub-objective. (See Tab. 1)

(2) Sub-objective 2: The minimum of resulted loss due to shortage of water supply

The sub-objective 2 is a target of the public effect because the shortage of water supply will affect social stability and production efficiency. This sub-objective effect represents importance degree of user's water demand in national economy. (See Tab. 2)

(3) Sub-objective 3: Satisfying the lowest demands of water for each water user

Each user has the lowest water demand. The better the lowest demand of water is satisfied.the better the planning project of water supply is. We select the lowest satisfied accuracy to describe the effect of this sub-objective.

We illustrate the hierarchical structure of the objective function of water allocation planning in Liao-He river basin in figure 1. These sub-objectives are interacted and constrainted each other. They are decided by experts in AHP to give undetermined subjective factor. The grey analytic hierarchy process discussed in this paper provides an effection measure method for computing the comprehensive effect of the objective.

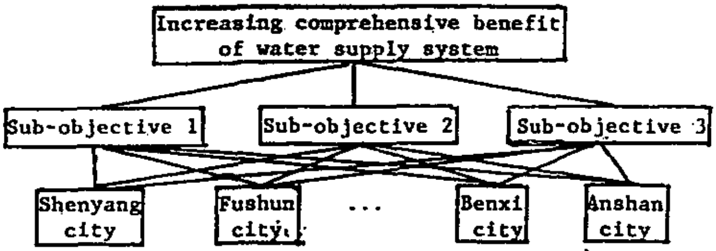

Fig.1. The Hierarchical Structure of the Objective Function

3. THE GREY ANALYTIC HIERARCHY PROCESS

The comprehensive effect of objective function is presented by the comprehensive benefit per unit volume water. These values which are calculated by using AHP show the user's weights.

In Saaty's hierarchical analysis, an expert is asked to supply ratios a_{ij} for each pairwise comparison between users for each sub-objective in a hierarchy. If A is a positive reciprocal matrix whose entries are a_{ij}. Saaty's procedure uses the pairwise comparison matrix A for each sub-objective to compute the final set of weights. In a complex system, it is difficult to obtain the ratioes a_{ij} by expert's judgement. The strategy method of the grey system theory find its application in AHP. The entries of positive reciprocal matrix are calculated with the grey strategy method. According to the Saaty's procedure, one first computes the weight, then obtains the comprehensive benefit.

(1) The method for computing effect measure

The grey strategy problem includes four elements, which are the event, the decision, the effect and the objective [3]. The combination of the event and the decision is called the game. Each game has an effect for the objective. The grey analytic hierarchy process is the method which the comprehensive effect measure of the game for objective is obtained by considering the effect of the game for each sub-objective. The entries of the positive reciprocal matrix A consist of the comprehensive effect measure of each game for the objective.

Let s_{ij} be the game element which is composed of the ith event and jth decision. $U_{ij}^{(p)}$ is denoted as the effect value of the game s_{ij} for the pth sub-objective. (i=1, ..., n. j=1, ..., m.). The effect measure of the game s_{ij} for the pth sub-objective is obtained by one of the following methods according to the feature of this sub-objective.

(a) The upper effect measure

$$r_{ij}^{(p)} = \frac{U_{ij}^{(p)}}{\mathrm{Max}\ \mathrm{Max}\ U_{ij}^{(p)}} \qquad i=1, \ldots, m. \quad j=1, \ldots, n. \quad (1)$$

(b) The lower effect measure

$$r_{ij}^{(p)} = \frac{\mathrm{Min}\ \mathrm{Min}\ U_{ij}^{(p)}}{U_{ij}^{(p)}} \qquad i=1, \ldots, m. \quad j=1, \ldots, n. \quad (2)$$

(c) The centre effect measure

$$r_{ij}^{(p)} = \frac{\text{Min } (U_{ij}^{(p)}, U_0^{(p)})}{\text{Max } (U_{ij}^{(p)}, U_0^{(p)})} \qquad i=1, \ldots, m. \quad j=1, \ldots, n. \quad (3)$$

Where: $U_0^{(p)}$ is assigned standard value.

We select the upper effect measure when the objective function takes maximum, and select the lower effect measure when the objective function is minimum. If the value of the objective function needs to take assigned standard value, we will select the centre effect measure.

By using the method discussed in this paper, we can get the effect measure matrix $R^{(p)}=\{r_{ij}^{(p)}\}_{m\times n}$, for the pth sub-objective ($p=1, \ldots, k$). Let $W=(w_1, \ldots, w_k)^T$ be the weighting value of each sub-objective in the objective. The comprehensive effect measure matrix $R=\{r_{ij}\}_{m\times n}$ is obtained as follows:

$$r_{ij}=\sum_{i=1}^{p} w_i \quad r_{ij}^{(p)} \qquad i=1, \ldots, n. \quad j=1, \ldots, m \quad (4)$$

(2) The ratios for each pairwise comparison

In the grey analytic hierarchy process, the entries a_{ij} of the reciprocal matrix $A=\{a_{ij}\}_{h\times h}$ ($h=n*m$) are shown by ratios between the comprehensive effect measure of each game for the objective. That is:

$$a_{ij}=\frac{r_i}{r_j} \qquad i=1, \ldots, h. \quad j=1, \ldots, h \quad (5)$$

(3) The properties of the positive reciprocal matrix satisfies the Saaty's rule

(a) The symmetric: $a_{ii}=1 \quad a_{ij} x a_{ji}=1$

(b) The transitive: $a_{ik} x a_{kj}=a_{ij}$, for i, j, k.

It is proved that the eigenvector corresponding to the largest eigenvalue of the positive reciprocal matrix can be defined as weight estimated value because the transitive of the positive reciprocal matrix A is satisfied.

(4) The method for computing weighting value of each game

The weighting value of each game can be obtained by using root method in AHP.

(a) Computing the product of the entries in the ith row of matrix A

$$M_i=\prod_{j=1}^{h} a_{ij} \qquad i=1, \ldots, h \quad (6)$$

(b) Computing M to the power of 1/h

$$W_h = (M_h)^{1/h} \qquad (7)$$

(c) Computing weighting value of each game

$$W_i' = \frac{W_i}{\sum_{i=1} W_i} \qquad (8)$$

(5) The comprehensive benefit per unit water

In the planning of water supply system, the game is denoted by the water user. So the comprehensive benefit per unit water of the user is the weight value in the grey AHP. That is:

$$C_i = W_i' \qquad i=1, \ldots, h \quad (9)$$

4. THE IMPLEMENTATION OF THE ALGORITHM

The algorithm in the grey AHP discussed in this paper is implemented on a IBM-PC micro-computer using FORTRAN language. The produce of the algorithm is as follows:

(1) Define the effect measure for each sub-objective;
(2) Compute the comprehensive effect measure for each objective;
(3) Form the positive reciprocal matrix;
(4) comuting weighting value of each game;
(5) Define the comprehensive benefit per unit water.

5. CONCLUSION

In this paper, we combine the strategy method of the grey system theory with AHP in order to develop Saaty's hierarchical analysis further. The grey analytic hierarchy process method can be used to supply ratioes a_{ij} by expert judgement. This method is effective in analysing system including imprecise factor and fuzzy relation. In the application deciding the objective function of the water allocation planning in Liao-He River basin, we obtain the comprehensive benefit per unit water of each user. The linear programming problem in the development planning of the water supply system is completely solved. It is obvious that the grey analytic hierarchy process method is a valuable implementation in practice.

This method has been tested in Liaoning province to study the optimum water supply allocation during 1987-2000 years. It will provide a scientific basis for the provincial governmental institution and may be useful to carry out relevant development planning project.

REFERENCES

Yi, Y. W. and Y. S. Zhang. (1987), "An Optimum Planning of Water Supply System in a Fuzzy environment", Working Paper of Shenyang Institute of Automation Academia Sinica.

T. Saaty, (1978), "Exploring the Interface Between Hierarchies Multiple Objectives and Fuzzy Set", Fuzzy Sets and Systems. 1.

J. L. Deng. (1987), Grey Forecasting and Decision Making. Huazhong University of Science and Technology, Press 1987.

J. J. Buckley, (1985), "Fuzzy Hierarchical Analysis", Fuzzy Sets and Systems. 17(3).

Liu, B. and S.B. Xu, (1984), "The Tools for Programming and Decision--AHP", Systems Engineering. (in Chinese). 6.