EVALUATING RELATIONSHIP OF CONSISTENCY RATIO AND NUMBER OF ALTERNATIVES ON RANK REVERSAL

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This paper shows a simulation result of rank reversal phenomenon with respect to the changing values of consistency ratio and number of alternatives. The simulation result reveals that number of alternatives and consistency ratio have significant effect to the occurrence of rank reversal. Numbers of alternatives of 4 and 5, and consistency ratio range between 0.02 and 0.10 are chosen to be investigated. Observation is based on rank reversal that occurs after adding a copy of best existing alternative.

As the initial phase, *n* order random reciprocal matrices are generated at desired C.R. interval. The C.R. interval range is from 0.02 to 0.10 and divided into smaller intervals that are: 0.02-0.04, 0.04-0.06, 0.06-0.08 and 0.08-0.10. The random reciprocal matrices contain random variables of the scale 1/9, 1/8, 1/7, ..., 1/2, 1, 2, ...,9, which are equally likely to occur. Then the priorities obtained are sorted to determine the rank order of alternatives. The next step is inserting a copy of best existing alternative to the reciprocal matrix. Again, the priorities are computed. For each random reciprocal matrix, the rank transition is recorded to construct a complete rank transition matrix. The elements of the rank transition matrix represent frequency of rank shifts for all random matrices that are produced. In particular, the rank transition matrix can be expressed as the following:

a_{11}	a_{12}	…	a_{1n}	i: the rank before a copy of alternative added
a_{21}	a_{22}	…	a_{2n}	j: the rank after a copy of alternative added
…	…	…	…	a_{ij}: number of rank transition from i to j
a_{i1}	a_{i2}	…	a_{nn}	

After a rank transition matrix is set up, the next computation is done to find the probability of total rank reversal occurrence. Let

$$S_i = \sum_{j=1}^{n} a_{ij} \quad \text{for } i = 1, 2\ldots, n \tag{1}$$

where S_i = sum of transition frequency from i-th rank

Probability of an alternative, which is previously in the i-th rank, moves to the j-th rank after adding of a copy of best alternative will be equal to a_{ij}/S_i and then expressed as p_{ij}. A rank transition probability matrix, which sum of its rows equals to 1, is produced as expressed below:

p_{11}	p_{12}	…	p_{1n}	i: the rank before a copy of alternative added
p_{21}	p_{22}	…	p_{2n}	j: the rank after a copy of alternative added
…	…	…	…	p_{ij}: probability of rank transition from i to j
p_{i1}	p_{i2}	…	p_{nn}	

The probability of total rank reversal occurrence can be computed by the following formula.

$$P_{klm} = \left(\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}\text{, where } i \neq j\right) / \left(\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}\right) = \left(\sum_{i=1}^{n}\sum_{j=1}^{n} p_{ij}\text{, where } i \neq j\right) / \left(\sum_{i=1}^{n}\sum_{j=1}^{n} p_{ij}\right) \quad (2)$$

where

P_{klm} = probability of total rank reversal occurrence at k-th row factor, l-th column factor and m-th replication.
k = row factor (number of alternatives) index {1, 2}
l = column factor (C.R. interval) index {1, 2, 3, 4}
m = replication index {1, 2, 3}

Two-way Analysis of Variance is employed to test whether there is significant effect of number of alternatives and C.R. interval towards the probability of total rank reversal occurrence (P_{klm}). The number of alternatives and C.R. interval are considered as factors, whereas the probability of total rank reversal occurrence (P_{klm}) serves as response variable. Table 1 represents the data structure of factorial experiment with 3 replication using a completely randomized design. In this paper, only relatively steady rank reversal probability is used.

Table 1. Data Structure of Factorial Experiment

	Consistency Ratio Interval			
Number of Alternatives	0.02-0.04	0.04-0.06	0.06-0.08	0.08-0.10
4	P_{111} P_{112} P_{113}	P_{121} P_{122} P_{123}	P_{131} P_{132} P_{133}	P_{141} P_{142} P_{143}
5	P_{211} P_{212} P_{213}	P_{221} P_{222} P_{223}	P_{231} P_{232} P_{232}	P_{241} P_{242} P_{243}

The result of two-way analysis of variance is shown below.

Two-way ANOVA: Probability of Rank Reversal Occurrence versus No. of Alternatives, C.R. Interval

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Analysis of Variance for Probability
Source        DF        SS        MS        F        P
No. of Alter   1 0.0027373 0.0027373   296.41    0.000
C.R. Interval  3 0.0011118 0.0003706    40.13    0.000
Interaction    3 0.0000677 0.0000226     2.44    0.102
Error         16 0.0001478 0.0000092
Total         23 0.0040645

                       Individual 95% CI
No. of Alter    Mean   ----------+---------+---------+---------+-
1            0.03802   (--*--)
2            0.05938                                    (--*--)
                       ----------+---------+---------+---------+-
                           0.04200   0.04800   0.05400   0.06000

                       Individual 95% CI
C.R. Interval   Mean   -------+---------+---------+---------+----
1             0.0406   (----*---)
2             0.0438         (---*---)
3             0.0528                        (---*---)
4             0.0576                                (---*---)
                       -------+---------+---------+---------+----
                         0.0420    0.0480    0.0540    0.0600
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Figure 1. Two-Way Analysis of Variance

The result shows that p-value of both factors is below 0.05. It substantiates the fact that there is significant effect of number of alternatives and C.R interval towards the probability of rank reversal occurrence. The larger the number of alternatives is the more likely the rank reversal occurrence. The larger the value of the consistency ratio, the more likely the for a rank reversal to occur. From the Analysis of Variance, it is also shown that these data do not provide enough evidence to claim that there is significant interaction effect at α=0.05.