PRACTICAL GROUP DECISION-MAKING METHOD USING INCOMPLETE PAIRWISE COMPARISON MATRICES FOR DIFFERENTIATED EVALUATION RESULTS

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Highlights

- A proposal for an incomplete pairwise comparison method that ensures perfect consistency in collective decision-making, along with its practical applications.
- A proposal for an incomplete pairwise comparison method designed to minimize the number of pairwise comparisons required from each evaluator, assuming multiple evaluators.
- The Analytic Hierarchy Process aimed at promoting public involvement in policy evaluation, precisely because we are in the era of AI.

ABSTRACT

The objective of this paper is to propose a practical Analytic Hierarchy Process (AHP) as a method for administrative evaluation based on citizen voting. When aggregating individual judgments or evaluations to make a collective decision, it is expected that the resulting outcomes will exhibit some degree of differentiation. To express such differentiation, the scales used in AHP can be utilized. However, when applying AHP in group decision-making, the challenge lies in constructing a pairwise comparison matrix that represents the group as a whole. The aim of this paper is to propose a method that simplifies the use of AHP for group decision-making by enabling the calculation of relative importance with the minimal number of comparisons, through preliminary ranking. The Harker's method is employed to estimate priorities from incomplete pairwise comparison matrices. The resulting pairwise comparison matrices are characterized by their perfect consistency, allowing all individual matrices, without adjustment, to be incorporated into a consistent group matrix. This method was devised as an evaluation approach for the administrative planning of a certain city in Japan and is primarily introduced in this paper as an application example.

Keywords: group decision-making method, Analytic Hierarchy Process, incomplete pairwise comparison matrix, perfect consistency, citizen-participatory evaluation.

1. Introduction

In organizational activities in Japan, "PDCA cycle" (Plan-Do-Check-Act), a business improvement framework, is emphasized as a key approach to promoting projects. This

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method, which originated in manufacturing for continuous quality improvement, involves planning, executing according to the plan, evaluating the results, and revising the plan based on the evaluation. Similar to organizational activities, the integration of the PDCA cycle into planning is also required in administrative activities.

When advancing administrative activities, it is expected to incorporate residents' opinions into the decision-making process, especially when determining the direction of policies and projects (hereafter referred to as "policies") based on their status. The challenge lies in how to aggregate the residents' opinions effectively to inform the direction of the policies. Moreover, when aggregating individual judgments or evaluations for collective decision-making, differentiation in the evaluation results is expected. As one approach to introducing such differentiation, this study considers utilizing the fundamental scales used in the Analytic Hierarchy Process (AHP), a method that can incorporate intuition and experience into group decision-making, making it suitable for such contexts.

The purpose of this paper is to formalize a method developed to evaluate priority policies in Okaya City's Commercial Revitalization Plan. The utility of this method is evidenced by its application over four annual iterations from FY2018 to FY2022 (excluding FY2019) as part of the PDCA cycle for the plan (see Appendices). The method's usefulness has been recognized by both the committee members and administrative staff involved in the evaluations (Iida, 2024), and its adoption was officially documented in the next Okaya City’s Commercial Revitalization Plan (FY2024 to FY2028) formulated in FY2023.

While this study includes the process of synthesis within the AHP framework, the example presented in this paper does not involve calculating overall evaluation values across the entire hierarchy due to the nature of the case. This work was supported by JSPS KAKENHI Grant Number JP20K01480.

2. Literature Review

The method proposed in this paper utilizes incomplete pairwise comparison matrices. A prominent research contribution in this area is Harker's method (Harker, 1987), which calculates importance values from incomplete pairwise comparison matrices. In this study, for an incomplete pairwise comparison matrix A, a non-negative matrix B is constructed by assigning 0 to undetermined pairwise comparison values and setting diagonal elements to the number of zeros in the corresponding row plus 1. The importance values are calculated using this matrix B. The consistency index for this case is also determined using the largest eigenvalue of B (Kułakowski, 2024).

In methods that use incomplete pairwise comparison matrices, when the number of elements is n, the first $n-1$ pairwise comparisons are of primary interest. On this issue, Wedley's study (Wedley, Schoner, & Tang, 1993) suggests conducting pairwise comparisons between the least important item and other items if the items are ranked. In the pairwise comparison table shown in Table 1, it is assumed that the items are arranged in ascending order of importance from left to right, and comparisons proceed in the order of (1), (2), (3), and (4). Alternatively, in the diagonal method, an alternative approach, comparisons are performed in the order of (1), (5), (8) and (10) in Table 1. The method examined in this paper is the latter order. However, the method proposed in this paper relies on only $n-1$ pairwise comparisons, making it independent of these prior studies.

Table 1. A pairwise comparison matrix for indicating pairs to be compared

	A1	A2	A3	A4	A5
A1	1				
A2	(1)	1			
A3	(2)	(5)	1		
A4	(3)	(6)	(8)	1	
A5	(4)	(7)	(9)	(10)	1

Harker and Wedley et al. have demonstrated in their respective studies that $n-1$ pairwise comparisons alone is not sufficient to accurately calculate importance values. The method proposed in this paper it redefines the relative evaluation values obtained by aggregating individual pairwise comparison matrices as the importance values of the corresponding items, with the goal of increasing the variability in relative evaluation values. As a result, pairwise comparison values exceeding 9 are also actively utilized.

To derive a group pairwise comparison matrix, Saaty strongly recommends using the geometric mean of individual pairwise comparison matrix elements (Saaty & Peniwati, 2007). This paper adopts this approach. In the calculation process, properties such as the invariance of eigenvector components to the reordering of items within the pairwise comparison matrix are also utilized.

This paper examines the application of Harker's method in the context of group decision-making by collecting individual pairwise comparison matrices. To the best of the author's knowledge, no prior research has addressed this perspective.

3. Objectives

The purpose of this paper is to formalize a method developed for evaluating the priority policies of the Okaya City Commercial Revitalization Plan. To begin, the paper explains the plan and the positioning of the importance of the priority policies.

Okaya City is a small industrial town with a population slightly below 50,000. In 2018, aiming to create a more livable city, the Okaya City Commercial Revitalization Plan was formulated, and a committee was established to promote it. This committee consisted of approximately 10 members, including representatives of local commercial unions, shop owners, and general citizens. The role of these members was to participate in the planning and evaluation of the priority policies and to operate the plan as a citizen-driven initiative through the PDCA cycle.

Committee members are required to determine the importance of policies. The importance values of policies to be ultimately determined are categorized as A, B, or C. Policies assigned an "A" are prioritized for budget allocation, and it was decided that 2 to 3 policies would be assigned an "A." While numerical outcomes such as cost-effectiveness and goal achievement are important, the intrinsic nature of administrative planning requires assessing whether the policies align with the fundamental goal of revitalizing commerce. Therefore, it can be said that subjective evaluations of which priority policies would most effectively revitalize the city's commerce, based on an understanding of the progress of current policies (or their initial state in the first round), are important.

To satisfy all these requirements, I considered the Analytic Hierarchy Process (AHP) for adoption. However, a significant challenge in applying AHP is the substantial increase in the number of pairwise comparisons as the number of evaluation targets grows. In this case, since general citizens were conducting the evaluations, the process needed to be simplified as much as possible to ensure its completion.

Specifically, this evaluation is not a one-time or large-scale survey conducted only once every few years, but rather one that needs to be carried out annually. Additionally, since some of the committee members serving as evaluators are periodically replaced, it cannot be expected that respondents will gradually become accustomed to the evaluation method. It is necessary to collect pairwise comparison values through a sufficiently simple survey.

On the other hand, the annual continuation of these evaluations is expected to provide numerical evidence for determining ABC classifications. Considering these factors, I proposed the following method.

STEP 1: The evaluators arrange the target policies in a single column in descending order of importance.
STEP 2: The evaluators start with the least important policy, conduct pairwise comparisons between adjacent policies. If there are n policies, this results in $n-1$ pairwise comparisons.
STEP 3: Using Harker's method to calculate the importance values based on the incomplete pairwise comparison matrix obtained in STEP 2. This effectively generates a complete pairwise comparison matrix for each individual.
STEP 4: From the multiple pairwise comparison matrices obtained in STEP 3, calculate a new group pairwise comparison matrix by taking the geometric mean for each element. This becomes the group pairwise comparison matrix.
STEP 5: Using the eigenvector method, calculate the importance values from the group pairwise comparison matrix obtained in STEP 4. These relative evaluation values represent the "importance" of each policy as evaluated by the group.

In the Okaya City Commercial Revitalization Plan, the overall evaluation corresponding to the integration stage of AHP was not calculated. The actual evaluation criteria consisted of two factors: "importance" and "urgency." However, since "urgency" in administrative contexts often requires special measures, integrating these criteria into a single index was deemed impractical. Therefore, this paper focuses solely on the evaluation criterion of "importance."

4. Methodology

The Okaya City Commercial Revitalization Plan consists of three fundamental strategies. Fundamental Strategy 1 includes six priority policies, Fundamental Strategy 2 includes three, and Fundamental Strategy 3 includes two. The primary focus of concern is Fundamental Strategy 1, which contains six priority policies (see Appendices).

While this paper does not address specific projects, each priority policy includes multiple projects aimed at achieving its objectives. The evaluation here is intended to determine which policies should be prioritized after understanding the progress of these projects, including the results presented in numerical form.

Below, the method for calculating the importance of these six priority policies is explained. To ensure consistency in evaluation, the same method was applied across all fundamental strategies, even for those with fewer policies, as differing evaluation methods by strategy were considered problematic. The question items used to construct the pairwise comparison matrix in STEPs 1 and 2are as follows:

(Question) Regarding the six policies (P1 to P6) under Fundamental Strategy 1: In the second column from the second row onward, list the symbols of the policies you consider more important in order. Then, in the cells of the third column, select the most appropriate adjective within the < > and circle it in order from the bottom.

Ranking	Policy	Circle the most appropriate adjective in < >.
1		<A little more / More / Much more>important than the 2nd place
2		<A little more / More / Much more>important than the 3rd place
3		<A little more / More / Much more>important than the 4th place
4		<A little more / More / Much more>important than the 5th place
5		<A little more / More / Much more>important than the 6th place
6		

Next, in STEP 3, the importance values are calculated using Harker's method based on the incomplete pairwise comparison matrix from the survey forms. The pairwise comparison values obtained from the survey correspond to (1), (5), (8), and (10) in Table 1 when arranged in ascending order of importance. Additionally, Table 2 is used for quantification.

Table 2. The fundamental scale used here

Definition of importance	Intensity
A little more important	3
More important	5
Much more important	7

In STEP 4, the individual importance values obtained in STEP 3 are combined into a single pairwise comparison matrix by calculating the geometric mean. While alignment of the order is necessary, this operation does not alter the importance values. Finally, in STEP 5, the importance values are determined using the eigenvector method based on the pairwise comparison matrix obtained in STEP 4. These values are normalized to represent the importance of each policy as evaluated by the group.

The evaluation results conducted using the method proposed in this paper from FY2018 to FY2022 (excluding FY2019) are presented below. These evaluations are based on the progress of the measures and are intended for the subsequent fiscal year. Therefore, the fiscal year in which the evaluation results are adopted is the year following the evaluation. FY2018 was the year in which the Okaya City Commercial Revitalization Plan was formulated, and since the plan had been implemented for only a short period, no evaluation was conducted in FY2019. Instead, the results from FY2018 were adopted as is.

Each survey period lasts for two to three weeks. Initially, the survey was conducted by sending and collecting survey forms via mail, but since 2021, it has also included online responses. The respondents consist of approximately 10 members of the committee (excluding the author), and the response rate has consistently been 100%. Notably, no confusion has arisen regarding the implementation of the surveys.

For example, from the row for Fundamental Strategy 1 in 2018 in Table 3, it can be observed that the ranking of importance is P2 > P1 > P5 > P6 > P3 > P4. Additionally, numerical differences can also be confirmed. Since there are six policies, a value exceeding 1/6 = 0.17 serves as one criterion. Based on the values obtained in Table 3, P1 and P2 were ultimately classified as "A."

Table 3. Transition of "Importance" of policies (FY2018 and FY2020 to FY2022)

	Fundamental Strategy 1						FS 2			FS 2	
FY	P1	P2	P3	P4	P5	P6	P1	P6	P7	P8	P9
2018	0.296	0.552	0.026	0.005	0.081	0.040	0.643	0.194	0.163	0.521	0.479
2020	0.712	0.144	0.015	0.005	0.006	0.118	0.551	0.341	0.108	0.740	0.260
2021	0.547	0.289	0.031	0.003	0.014	0.117	0.347	0.426	0.227	0.682	0.318
2022	0.443	0.356	0.068	0.011	0.041	0.080	0.252	0.556	0.192	0.742	0.258

Table 4. Transition of "Urgency" of policies (FY2018 and FY2020 to FY2022)

	Fundamental Strategy 1						FS 2			FS 2	
FY	P1	P2	P3	P4	P5	P6	P1	P6	P7	P8	P9
2018	0.097	0.784	0.028	0.004	0.051	0.036	0.542	0.284	0.174	0.619	0.381
2020	0.087	0.354	0.040	0.007	0.035	0.478	0.423	0.448	0.129	0.626	0.374
2021	0.288	0.447	0.021	0.013	0.059	0.173	0.289	0.499	0.212	0.770	0.230
2022	0.254	0.326	0.066	0.007	0.055	0.291	0.142	0.673	0.185	0.729	0.271

5. Methodology Analysis

The method formalized in this paper has three primary issues. The first is the requirement for ranking in STEP 1. The second is that in STEP 2, pairwise comparisons are conducted only between adjacent items, and the Harker's method is applied. The third issue is that, in the case study introduced in Section 4, tied rankings are not utilized, and the values 3, 5, and 7 are applied. This section examines these issues in detail.

Firstly, the ranking in STEP 1 is a commonly used method in practical applications and for determining preference orders. However, AHP acknowledges the existence of items that cannot necessarily be ranked. In this case study, since residents were required to take responsibility for expressing their judgments, it was assumed that evaluators would be capable of ranking the items. In practice, no issues related to this assumption have been observed.

Next, in STEP 2, it is common to assign scores in descending order of importance, such as $n-1$, ..., 2, and 1 points. However, this method may not necessarily result in significant

differences in the final evaluation values. Therefore, in this case, the range between policies was measured using the three options provided in Table 2. Nevertheless, as seen in Section 2, from the perspective of AHP, the data from $n-1$ comparisons cannot be expected to achieve precise estimation accuracy. However, since the $n-1$ comparisons here are based on a prior ranking, there is no risk of rank reversal. In this way, the relative evaluation values derived through this process were redefined as the importance of the policies.

As seen in Section 2, an alternative approach for STEP 2 could involve using the "least important item as a reference point for comparisons." While this method may improve estimation accuracy, it has limitations in differentiating items depending on the number of comparison items. For example, with four intensity levels, it is not possible to differentiate more than six items.

Finally, the reason why the AHP feature of "equally important" was not adopted in this case study lies in the objective of differentiating evaluation results. It is also known that respondents in general surveys tend to choose neutral options such as "neither." In this case, if many evaluators refrain from making decisions and select "equally important," the evaluation results become unusable. In questionnaire surveys where retries are not possible, it is crucial to obtain the expected results in a single attempt.

The case study in this research was presented at ISAHP 2022 and 2024. The calculation method introduced there included the following steps to derive individual importance values from the survey results and calculate the group importance as a committee. Here, the term "projects" will be used as is.

(Calculation Procedure 1) Calculation of Individual Importance for Each Project
First, assign an importance value of 1 to the project ranked lowest (in this case, 6th place). Then, for a project ranked n with an importance value of s, the importance of the project ranked $(n-1)$ is calculated by multiplying s by a factor of k. Here, k is a value derived from the survey results and converted according to Table 2. Repeat this process to calculate the importance values for all projects up to the 1st place.

(Calculation Procedure 2) Calculation of Project Importance as a Committee
The importance of each project as determined by the committee is calculated using the geometric mean of individual importance values. Finally, these values are normalized to determine the final importance of each project, ensuring that the total importance across all projects equals 1.

Subsequent research demonstrated that the calculation method described above is equivalent to the process outlined in STEP 1 to STEP 5. By focusing on STEP 1 to STEP 5 from the perspective of AHP and applying equivalent transformations to the calculation process, it was shown that the method relies solely on simple ratios and geometric means. This makes the final results and the calculation process intuitive and easy to understand for evaluators.

6. Conclusions

The purpose of this paper was to formalize a method developed for practical use in administrative settings. This method has functioned ideally in the administrative context

and was officially adopted for the second-phase plan that began in the fiscal year 2024. Since fiscal year 2022, in addition to using paper-based questionnaires, responses via web pages have been incorporated. This has enabled a greater number of evaluators to participate in the evaluation process.

The use of AHP in group decision-making in administrative activities contexts remains limited, at least in Japan. Challenges include the occurrence of inconsistent pairwise comparison matrices and the difficulty general citizens face in conducting numerous pairwise comparisons. These challenges could potentially be resolved by devising innovative approaches to structuring questionnaire questions as shown in this paper. On the other hand, with the widespread adoption of pairwise comparisons in AHP, it is expected that the AHP with a complete pairwise comparison matrix will be utilized to estimate the potential value of policies.

7. Limitations

The method proposed in this paper is not suitable for pairwise comparisons based on intuitive responses. It requires careful consideration of the context in which it is applied. In practice, it is necessary to perform an initial ranking. For example, when applying this method to policy evaluations by citizens, it is essential to first explain the content of the policies and facilitate sufficient discussion and exchange of opinions.

In the case study presented in this paper, a meeting was held prior to the survey. It is inherently inappropriate for individuals tasked with policy evaluation to make intuitive judgments when they do not fully understand the content of the policies or do not have an accurate grasp of the current situation. Evaluators are expected to take responsibility for expressing their own opinions as individuals who have a thorough understanding of the plan's policies and the progress of the projects. Users of this method will need to devise appropriate measures for these operational aspects.

8. Key References

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9. Appendices

The Okaya City Commercial Revitalization Plan consists of three fundamental strategies and nine kind of priority polices. As shown below, Fundamental Strategy 1 includes six priority policies (P1, P2, P3, P4, P5 and P6), Fundamental Strategy 2 includes three (P1, P6 and P7), and Fundamental Strategy 3 includes two (P8 and P9). P1 and P6 are included in Fundamental Strategies 1 and 2, but these are different measures, even though they share the same name, because they are based on different perspectives.

Fundamental Strategy 1: Strategy for Enhancing the Appeal of Shopping Streets (Initiatives for Local Stores, Shopping Streets, and Commercial Associations)

P1: The Role of Shopping Streets (Responding to the Diversification of Consumer Values in Shopping Streets). Keywords: A City Where People Gather Triggered by Shopping, Initiatives Viewing the City as a Single Mall ("Okaya Mall"), Support for Child-Rearing Households, Support for Seniors, and Support for Shopping-Disadvantaged Individuals.

P2: Support for Capital Investment to Enhance the Appeal of Commerce. Keywords: Enhancing Store Appeal, Adapting to Cashless Payment Systems.

P3: Support for Capital Investment to Enhance the Appeal of Commerce. Keywords: Pre-Startup Support, Post-Startup Support, Effective Information Dissemination, and Attracting Aspiring Entrepreneurs to the City.

P4: Initiatives to Support Business Succession for Store Owners. Keywords: Raising Awareness of Business Succession, Intra-Family Succession, Non-Family Succession.

P5: Consideration of Strategies for Addressing Vacant Stores. Keywords: Approaches to Information Dissemination, Initiatives to Enhance the Value of Vacant Stores.

P6: Promotion of Information Dissemination to Enhance the Recognition of Local Stores. Keywords: Continuous Information Dissemination, Information Sharing via Online Advertising and Social Media (SNS).

Fundamental Strategy 2: Strategy for Enhancing Consumer Satisfaction

P1: The Role of Shopping Streets (Responding to the Diversification of Consumer Values in Shopping Streets). Keywords: Changes in Consumer Behavior, Diversification of Needs.

P6: Promotion of Information Dissemination to Enhance the Recognition of Local Stores. Keywords: Adapting to Diverse Information Acquisition Methods

P7: Initiatives to Attract Customers from Outside the City. Keywords: Expanding the Market Area Beyond the City, Expanding Collaborative Organizations.

Fundamental Strategy 3: Commercial Infrastructure Development Strategy

P8: Initiatives to Promote Collaboration Among Stakeholders for Commercial Revitalization/ Initiatives to Encourage Active Participation of Local Businesses in Commercial Activities/ Initiatives to Promote Events that Contribute to Commercial Development. Keywords: None.

P9: Consideration of the Role and Structure of Commercial-Related Organizations in the City. Keywords: None.

The following is a line graph representing the transitions of the numerical values presented in Tables 3 and 4. It can also be observed that the values fluctuate moderately. Notably, in FY2020, there was significant fluctuation due to the impact of the COVID-19 pandemic.

Figure 1. Fundamental Strategy 1 (FY2018 and FY2020 to FY2022)

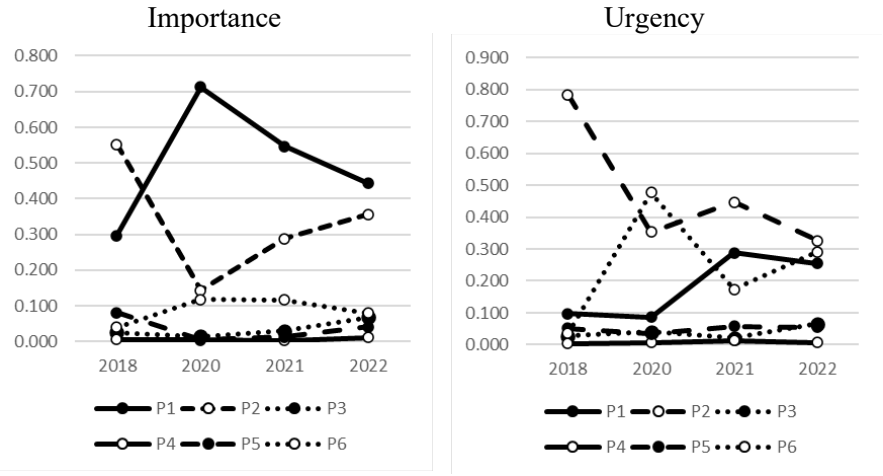

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Figure 2. Fundamental Strategy 2 (FY2018 and FY2020 to FY2022)

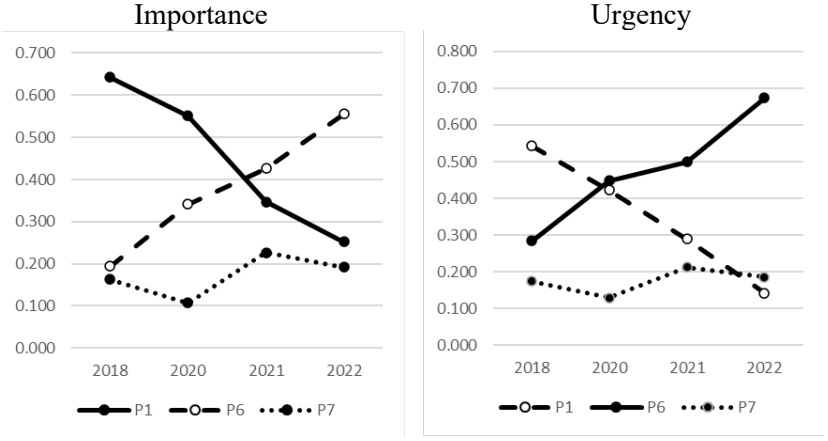

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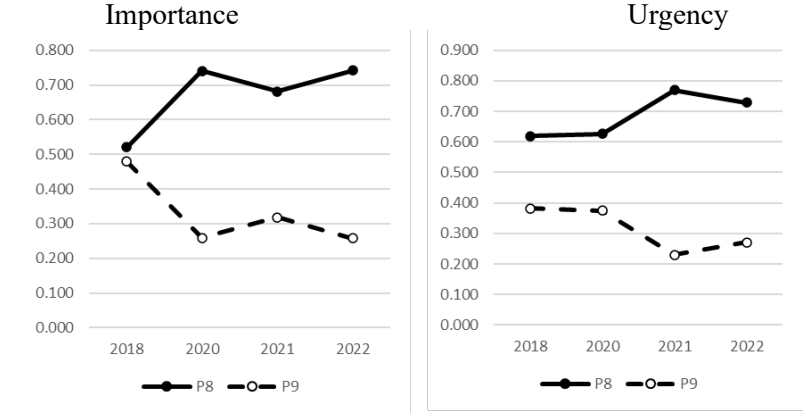
Figure 3. Fundamental Strategy 3 (FY2018 and FY2020 to FY2022)



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