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AHP GROUP DECISION MAKING AND CLUSTERING

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1 Introduction

The application of the Analytic Hierarchy Process (AHP) for group decisions is well documented (Saaty and Vargas, 2007). Within this study, we will provide a comprehensive overview over AHP group decision making. The core research goal is the systematic identification of homogenous groups which is of special interest if we have a large number of decision makers. We will present a simple but effective methodology on clustering AHP group evaluations. Within our approach no modification of AHP theory is necessary as suggested by other authors (e.g. Song and Hu, 2009) in order to identify homogenous groups of decision makers. If we have only a small group of decision makers, the identification of similarities and differences between the individual evaluations is easy. The analysis of the heterogeneity can be done intuitively. However, if the number of decision makers rises, this intuitive way of building groups of "similar" evaluations gets more and more difficult. A better way of grouping individual judgments will be presented within this contribution.
Saaty's fundamental scale (or any other scale) can be used for subjective evaluations of decision makers; the usual (deterministic) approach of approximating weights out of pairwise comparisons is used, the eigenvector method (Basac and Saaty, 1993). As a consequence, geometric means method will be used to aggregate individual decisions (Wu et al., 2008). To identify decision makers with similar evaluations, a well-established distance measurement in statistics is applied (Squared Euclidian Distance).
We will present a numerical example evaluating a simple decision hierarchy. The outcome of this experimental data clearly shows that the proposed AHP clustering methodology can be easily applied with large numbers of decision makers. The (computer aided) analysis is done when the importance of individuals within the group is equivalent. Other authors discussed aggregation methodologies where importance of individuals is non-equivalent (e.g. Beynon, 2005). However, this can be considered to be illusive if we have a large number of decision makers (e.g. opinion polls) where the influence of each individual on the final decision is usually equal. These evaluations are usually simplified by offering only yes-no pre-formulated answers. With our methodology, the identification of homogenous groups of

decision makers is simple. As a consequence, the evaluation of decision hierarchies instead of simple yes-no evaluations would help to get much deeper insights into the true opinions of communities.

2 Group decision making and the AHP

Concerning literature (Altuzarra et al., 2007) there are two principal possibilities of AHP grouping:[1] (1) aggregation of individual judgments and (2) the aggregation of individual priorities (aggregating the approximated priorities of decision makers). Both cases should deliver more or less comparable results (in case of more or less consistent evaluations). However, as group decisions usually are made in a way that individual judgments are not accessible for further interpretation, possibility (2) – where pairwise comparison matrices are aggregated – seems to be the more relevant way of aggregation. Because of the axioms of AHP (Saaty, 1980), only reciprocal pairwise comparison matrices fulfilling the condition

$$a_{ij} = \frac{1}{a_{ji}} \quad (1)$$

are acceptable. Therefore, the geometric mean is used to aggregate pairwise comparisons.

However, principal questions arise when larger groups of decision makers deliver evaluations: What if the individual judgments differ significantly? Can an aggregated result then represent a reliable consensus representing all individual judgments? The last question can definitely be negated. Only think about the following simplified case, where two decision makers deliver a pairwise comparison for element i and j. The first one evaluates $a_{ij} = 9$, the second one $a_{ij} = {}^1/_9$ (the absolute opposite). In this extreme case, the aggregated evaluation would be $a'_{ij} = 1$ representing none of the original evaluations. Therefore, we propose in the following section an easy and reliable way of including the *distance* between individual evaluations using a common way of distance metrics. This is especially relevant if we have a larger number of decision makers and complex decision hierarchies, where the individual differences cannot be assessed intuitively.

3 AHP Clustering

To present our principle clustering approach we use a numerical example consisting of evaluations of 20 decision makers (dm_i). The decision hierarchy consists of 5 attributes (element E_1 to E_5). The priorities $w(E_i)$ of the individual evaluations of this hierarchy can be taken from Table 1. The experimental data[2] are all consistent with CR<0,1 (Saaty, 1980). Further information concerning the decision hierarchy and subjective evaluations can be found at Meixner (2009).

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[1] We leave out other possibilities of getting group decisions where e.g. the "group first unanimously agrees upon criterion weights" (Van Den Honert and Lootsma; 1996: they mention different possibilities of getting a group consensus).

[2] The data were randomly selected from a larger sample (Meixner, 2009) to prove the principal applicability of the presented approach.

Table 1: Individual priorities of dm_i

Decision maker	Priorities of attributes					CR
	$w(E_1)$	$w(E_2)$	$w(E_3)$	$w(E_4)$	$w(E_5)$	
dm_1	0,129	0,146	0,384	0,099	0,241	0,049
dm_2	0,086	0,355	0,312	0,050	0,196	0,091
dm_3	0,256	0,150	0,460	0,046	0,087	0,067
dm_4	0,194	0,250	0,293	0,118	0,145	0,065
dm_5	0,126	0,212	0,283	0,149	0,231	0,021
dm_6	0,140	0,257	0,242	0,097	0,264	0,029
dm_7	0,248	0,301	0,201	0,041	0,209	0,083
dm_8	0,093	0,299	0,265	0,134	0,209	0,063
dm_9	0,273	0,390	0,251	0,047	0,040	0,047
dm_{10}	0,055	0,133	0,404	0,029	0,378	0,090
dm_{11}	0,067	0,284	0,491	0,064	0,094	0,022
dm_{12}	0,123	0,337	0,223	0,115	0,202	0,096
dm_{13}	0,119	0,146	0,377	0,065	0,292	0,063
dm_{14}	0,060	0,270	0,393	0,092	0,186	0,036
dm_{15}	0,153	0,237	0,281	0,135	0,195	0,027
dm_{16}	0,166	0,364	0,298	0,106	0,067	0,038
dm_{17}	0,157	0,305	0,276	0,101	0,161	0,036
dm_{18}	0,282	0,169	0,344	0,063	0,141	0,045
dm_{19}	0,064	0,202	0,360	0,135	0,239	0,040
dm_{20}	0,057	0,251	0,344	0,121	0,226	0,004

Distance metrics. Obviously, the individual priorities are quite comparable in some cases (e.g., dm_1 and dm_{13}), and sometimes differ significantly (dm_1 and dm_7). In other words, the *distance* between dm_i and dm_j is quite low in some cases and large in others (where $i \neq j$).

For this purpose, it is necessary to measure the distance between two subjects. There are different approaches available to approximate the distance between two clusters. We used the Squared Euclidian Distance, but any other distance measurement could be taken as well (Manhatten, Euclidian Distance, etc.). Further, similarity measure of two subjects could be an adequate approach, too (Pearson Correlation, Spearman Correlation, etc.).

The general formula using the Squared Euclidian Distance $d_{\min}$ with m elements and n decision makers is:

$$d_{\min} = \min \sum_{k=1}^{m} \left[w_i\left(E_k\right) - w_j(E_k) \right]^2 \quad \text{for all } i, j = 1 \ldots n,\ i \neq j \tag{2}$$

In a first step, the minimum distance $d_{\min}$ can be found between dm_5 and dm_{15} with $d_{\min} = 0.0028060897308$ (the maximum distance amounts to 0.25168).

Table 2: Initial distance matrix

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	0.055																		
3	0.048	0.105																	
4	0.033	0.030	0.050																
5	0.017	0.034	0.083	0.015															
6	0.033	0.024	0.107	0.020	0.008														
7	0.076	0.042	0.105	0.024	0.042	0.021													
8	0.041	0.012	0.109	0.018	0.010	0.009	0.037												
9	0.141	0.064	0.104	0.044	0.101	0.088	0.040	0.077											
10	0.030	0.092	0.129	0.107	0.062	0.066	0.135	0.088	0.252										
11	0.057	0.048	0.055	0.062	0.078	0.098	0.131	0.070	0.114	0.113									
12	0.064	0.014	0.127	0.021	0.021	0.011	0.023	0.005	0.057	0.117	0.092								
13	0.004	0.058	0.068	0.048	0.024	0.033	0.079	0.048	0.163	0.014	0.074	0.071							
14	0.023	0.016	0.069	0.031	0.025	0.036	0.076	0.021	0.103	0.060	0.019	0.038	0.031						
15	0.023	0.027	0.070	0.005	0.003	0.008	0.029	0.008	0.071	0.080	0.069	0.015	0.033	0.024					
16	0.086	0.026	0.084	0.020	0.054	0.054	0.045	0.031	0.019	0.179	0.056	0.027	0.108	0.043	0.034				
17	0.044	0.013	0.076	0.005	0.017	0.014	0.020	0.007	0.039	0.109	0.061	0.007	0.055	0.025	0.007	0.013			
18	0.037	0.077	0.018	0.020	0.046	0.055	0.044	0.069	0.068	0.114	0.083	0.075	0.051	0.065	0.034	0.061	0.041		
19	0.009	0.035	0.080	0.033	0.010	0.025	0.079	0.020	0.138	0.037	0.050	0.043	0.014	0.010	0.017	0.071	0.034	0.064	
20	0.018	0.019	0.088	0.028	0.011	0.019	0.066	0.010	0.115	0.049	0.043	0.027	0.023	0.005	0.015	0.052	0.022	0.068	0.003

Aggregation. In a next step, this two subjects will be grouped. As mentioned above and shown by Aczél and Saaty (1983), the best aggregation procedure is the geometric mean. The pairwise comparison matrix $\boldsymbol{A}_5$ and $\boldsymbol{A}_{15}$ [3]

$$\boldsymbol{A}_5 = \begin{pmatrix} 1 & 0.83 & 0.33 & 0.77 & 0.55 \\ 1.2 & 1 & 1 & 1.5 & 0.91 \\ 3 & 1 & 1 & 1.5 & 1.5 \\ 1.3 & 0.67 & 0.67 & 1 & .5 \\ 1.8 & 1.1 & 0.67 & 2 & 1 \end{pmatrix}, \boldsymbol{A}_{15} = \begin{pmatrix} 1 & 0.67 & 0.5 & 1 & 1 \\ 1.5 & 1 & 1 & 2 & 1 \\ 2 & 1 & 1 & 3.1 & 1 \\ 1 & 0.5 & 0.32 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

are aggregated to $\boldsymbol{A}_{5,15}$:

$$\boldsymbol{A}_{5,15} = \left(\prod \boldsymbol{A}_5, \boldsymbol{A}_{15}\right)^{0.5} = \begin{pmatrix} 1 & 0.75 & 0.41 & 0.88 & 0.75 \\ 1.34 & 1 & 1 & 1.73 & 0.95 \\ 2.45 & 1 & 1 & 2.16 & 1.23 \\ 1.14 & 0.58 & 0.46 & 1 & 0.71 \\ 1.34 & 1.05 & 0.82 & 1.41 & 1 \end{pmatrix}$$

Priorities are approximated using the Eigenvector method (Saaty, 1980) and a new distance matrix is calculated where dm_5 and dm_{15} are now represented by cluster $dm_{5,15}$ (Table 3).

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[3] Because of the nature of the original study (Meixner, 2009) the respondents evaluating the hierarchy had the possibility to use a real interval scale from 1 to 9 (1 to 1/9) with any intermediate values (e.g. 1.23) based on the AHP fundamental scale (Saaty, 1980) instead of only choosing between integer numbers 1, 2, 3, 4, 5, 6, 7, 8, 9. The respondents had therefore the possibility to assess even more accurate evaluations; the semantic meaning of the scale was kept.

Table 3: Aggregated weights $w(E_i)$

	$w(E_1)$	$w(E_2)$	$w(E_3)$	$w(E_4)$	$w(E_5)$	CR
dm_5	0.126	0.212	0,283	0,149	0,231	0.021
dm_{15}	0,153	0,237	0,281	0,135	0,195	0.027
$dm_{5,15}$	0,140	0,223	0,281	0,141	0,212	0.008

Full Clustering. This process is repeated until all subjects dm_i are aggregated into one cluster. If more than two subjects are aggregated, the initial pairwise comparison matrices are aggregated by use of the geometric mean. E.g., in step 7 group $dm_{5,15}$ and dm_6 have the minimum distance d_{min}. $\boldsymbol{A}_5$, $\boldsymbol{A}_{15}$, and $\boldsymbol{A}_6$ are aggregated to $\boldsymbol{A}_{5,6,15}$

$$\boldsymbol{A}_{5,6,15} = \left(\prod \boldsymbol{A}_5, \boldsymbol{A}_6, \boldsymbol{A}_{15}\right)^{1/3} = \begin{pmatrix} 1 & 0.64 & 0.49 & 0.15 & 0.57 \\ 1.56 & 1 & 1 & 1.79 & 1,06 \\ 2.03 & 1 & 1 & 2.38 & 1.14 \\ 0,87 & 0.56 & 0.42 & 1 & 0.59 \\ 1.75 & 95 & 0.87 & 1.69 & 1 \end{pmatrix}$$

Error measurement. In order to select the "right" number of clusters where
(1) information loss is acceptable and
(2) error within clusters is low while the distance between clusters is high
it is necessary to measure the error occurring when two groups are accumulated. In our opinion an easy and appropriate way of evaluating the clustering error is the following: we simply sum up all distances between subjects that are grouped. Adding up the distances results in the total sum of distances (TSD) which will increase during all aggregation steps (unless evaluations of dm_i and dm_j are totally equal).

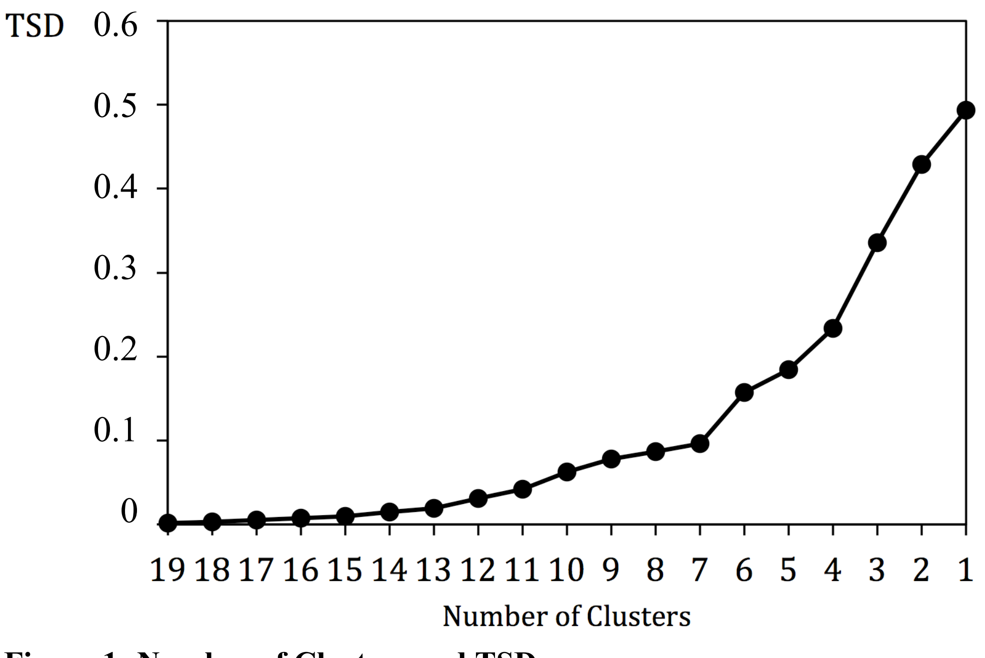

Figure 1: Number of Clusters and TSD

Selection of number of clusters. It is now up to the researcher to select the most appropriate number of clusters by using TSD (Figure 1). Usually, one will select a cluster solution *before* a significant rise of TSD. TSD is significantly increasing from 7 to 6 clusters and then from 4 to 3, from 3 to 2 and when all subjects are aggregated into one cluster. In our case, we further analyze only the outcome of the 4-cluster solution. However, this is up to the researcher and it is a tradeoff between minimum information loss (TSD) and the chance for a reliable and appropriate interpretation of clusters.

As we can see from the AHP clustering dendogram (Figure 2) where the cluster steps and the related TSD (horizontal connecting lines) are visualized, we get one big cluster with subjects $dm_{5,15,6,4\ \ldots 11}$ and 3 small clusters $dm_{7,9,16}$, $dm_{3,18}$, and dm_{10}. At least the last "cluster" consisting only of dm_{10} may be considered to be an outlier.

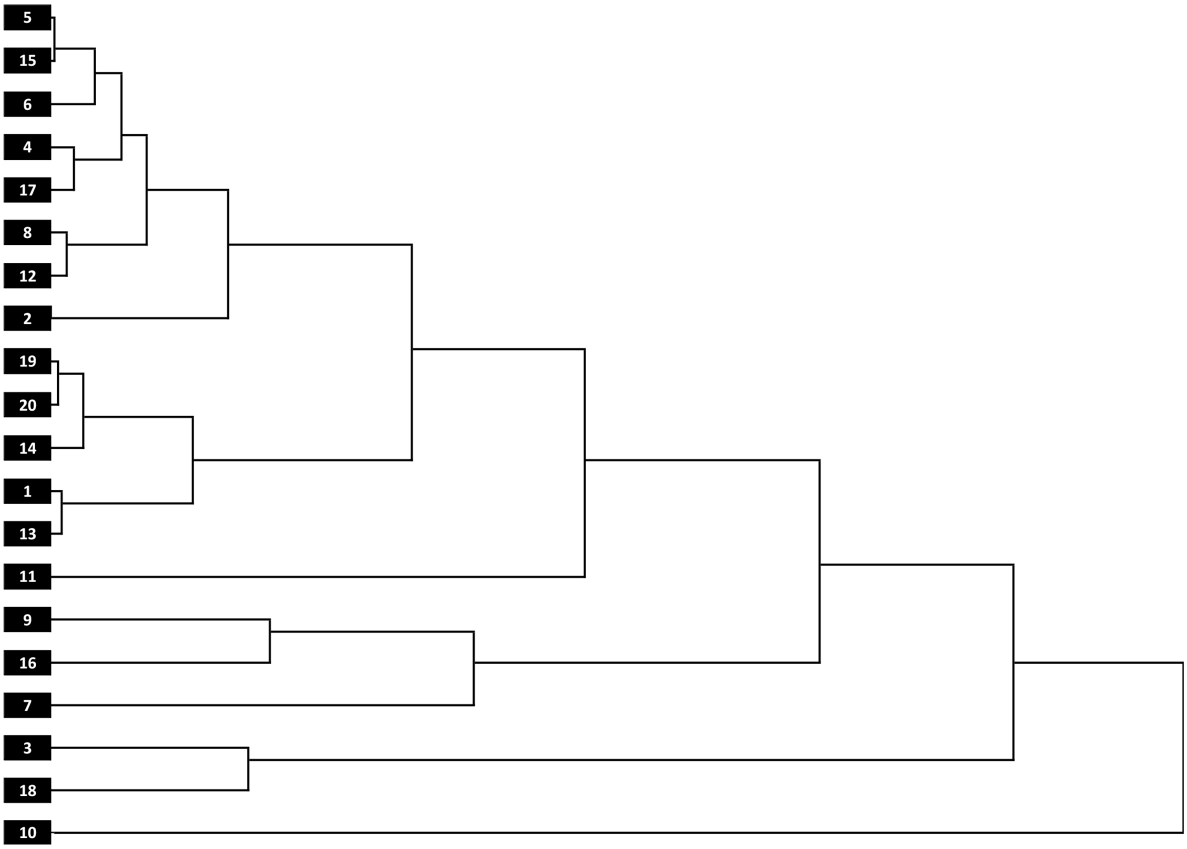

Figure 2: AHP clustering dendogram

Approximation of weights for cluster 1 to 4 show that there are considerable differences between the clusters. For cluster 1 E_3 seems to be of special importance, while cluster 2 mainly reflects on E_2. Priorities for all clusters and the complete aggregation can be taken from Table 4. TSD amounts to a maximum of 0.188 for Cluster 1 compared to 0.494 if all subjects were aggregated into one group.

Table 4: Cluster analysis – approximation of $w(E_i)$, CR and TSD

	$w(E_1)$	$w(E_2)$	$w(E_3)$	$w(E_4)$	$w(E_5)$	CR	TSD
Cluster 1	0,107	0,255	0,329	0,104	0,205	0,002	0,188
Cluster 2	0,233	0,367	0,255	0,062	0,083	0,012	0,037
Cluster 3	0,275	0,159	0,401	0,055	0,111	0,035	0,009
Cluster 4	0,055	0,133	0,404	0,029	0,378	0,090	0,000
Complete aggregation	0,133	0,259	0,340	0,088	0,181	0,002	0,494

Figure 3 visualizes the differences between the clusters: $w(E_1)$ compared with $w(E_2)$. It clearly shows that the evaluations of the total group of 20 decision makers are by far not homogenous. If we have a large number of decision makers, it might be wise to analyze the sample with an appropriate method. The AHP Cluster Analysis proposed herein could deliver more insights into the real meaning of the subjects' evaluations as we get relatively homogenous subgroups.

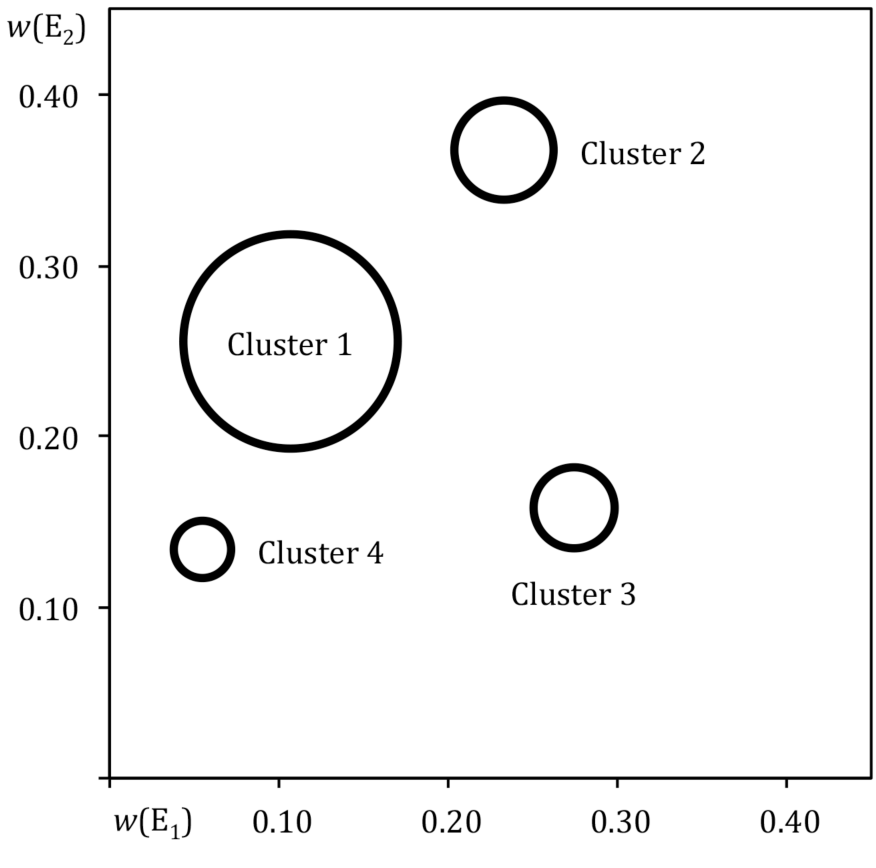

Figure 3: 4 Cluster solution – $w(E_1)$ vs. $w(E_2)$

4 Discussion and Summary

This contribution described the analytical steps to group individual AHP evaluations into homogenous subgroups. This might be especially necessary in decision situations where we have a large number of decision makers with considerable differing evaluations. In our numerical example, we used a data set of 20 experimental cases. The results showed, that a number of different clusters could be identified. The aggregated results per cluster differed significantly. A discussion and further analysis on why there are these differences and what are the consequences of this fact would complete the analytical results. In order to identify the difference between individual priorities of decision makers, we used the Squared Euclidean Distance. However, any other distance metrics could be used. An in-depth comparison with the present results could be an interesting future research application. Instead of distance metrics, similarity measures would be possible; in this case, maximum e.g. correlation must be searched within a similarity matrix. In all, we presented an easy to apply approach to analyze homogeneity within group decisions, which is of raising importance if the AHP is applied for group decisions with a large number of decision makers.

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